Druckexemplar

~~DESCRIPTION~~

~~"THREE BARS KIT TO BUILD POLYHEDRIC STRUCTURES BASED ON THE LAW OF COSINES"~~

Field of the invention

The main field of this invention is mechanical engineering where a kit of bars constitutes engineering elements for construction of generic structures.

The structures have elongated members which are designed for the purpose of being joined to similar members in various relative positions.

The object of this invention is an engineering structure construction system that simultaneously maximizes structure diversity and minimizes the system complexity.

Background of the invention

One must say that the CONSTRUCTION KIT of document WO2004024277 by John Warner Timothy bears striking resemblance with this three bars kit but, nevertheless, the differences are clear:

1. document WO2004024277 is a toy whereas this three bars kit is meant for engineering structures;
2. claim 1 of document WO2004024277 covers unitary cubic blocks with a single hole whereas this three bars kit uses unitary cubic blocks with two holes;
3. document WO2004024277 makes no claim relative to ***H/L***;
4. "triple connections" and the law of cosines are not mentioned in document WO2004024277.

Another document worth mentioning is WO9921669 by Massimo Ferrante and Mario Amato as it deals with METAL OR NON

METAL SECTION BARS WITH "L", "U", "Z" SHAPED CROSS SECTIONS. The kind of bars mentioned in document WO9921669 differs from the present three bars kit in the absence of a geometric requirement for the thickness of the material. This means that the diversity of polyhedric structures which can be built with one of "L", "U" or "Z" shaped cross section bars is small when compared with the diversity of polyhedric structures which can be built with one of the three bars from the three bars kit.

Yet another related document is the UNIVERSAL CONSTRUCTION SYSTEM of document WO0045083 by Salvatore Mocciaro which is focused on parts manufactured by extrusion. In this case the main difference is that the bars of document WO0045083 are much more than three. It is true that a simplified manufacturing process may compensate additional costs when projecting and assembling a given structure but a complex system is, with a high probability, more expensive in general than an inherently simple and mathematically rigorous system like the three bars kit.

And, finally, it is important to mention the Portuguese patent PT103301 by the present author. Patent PT103301 is entirely general in respect to the types of polyhedric structures that can be built with it. This generality results from the fact that angles and lengths are selected from continuous ranges. Unfortunately, this generality comes at the cost of a need for iterative processes to reach the final form of the structures. In the absence of preformed gauges or shapes, the angles and the lengths start out wrong. This just cannot happen with the present three bars kit because angles and lengths are selected from a finite set of discrete values. The three bars kit gives total surety about matching dimensions together with good generality.

Summary of the invention

The present invention is a three bars kit constituted by three kinds of bars, where the first kind of bar has uniform square convex hull section along its length, and both the other two bars, the second and third kind of bars, have equal and uniform regular hexagonal convex hull sections along their lengths. The three bars provide the means to construct generic complex engineering structures that can be thought of as assemblies of simple sub-structures like: triple connections as shown in Figures 4, 5, 6 and 7; sheaves of bars mutually connected in parallel as shown in Figures 8, 9 and 10; polygons whose edges have integer length as shown in Figure 11; quadruple connections as shown in Figures 16 and 17; etc.

Brief description of the drawings

Figure 1: First kind of bar (1).

Figure 2: Second kind of bar (2).

Figure 3: Third kind of bar (3).

Figure 4: Triple connection of three bars of first kind.

Figure 5: Triple connection of two bars of first kind and one bar of second kind, the second kind of bar being inside the 120 degrees corner.

Figure 6: Triple connection of two bars of first kind and one bar of second kind, the second kind of bar being inside the 60 degrees corner.

Figure 7: Triple connection of three bars of third kind.

Figure 8: Three bars of third kind can be mutually connected in parallel (exploded view).

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Figure 9: Three bars of second kind can be mutually connected in parallel using only sides of 2x***L*** wave vector (exploded view).

Figure 10: Three bars of second kind can be mutually connected in parallel matching a pair of ***G***/2 wave vector sides by a reference hole on those sides (exploded view).

Figure 11: Example of a triangle with a 90 degrees corner.

Figure 12: Example of a structure made exclusively with the third kind of bars of kind.

Figure 13: Example of a structure made with first and second kind of bars.

Figure 14: Example of a connection between two bars with scales related by factor 1/3.

Figure 15: Example of a connection between two bars with scales related by factor 1/4.

Figure 16: The special quadruple connection.

Figure 17: Example of quadruple connection made exclusively with the third kind of bars.

Figure 18: Graph of results of the integer law of cosines for rectangle triangles (Pythagorean Theorem).

Figure 19: Graph of results of the integer law of cosines for triangles with a 60, 120, 109.47 or 70.53 degrees corner (respectively with cosines -1/2, 1/2, 1/3 or -1/3). Points are at an integer distance from the origin.

Figure 20: Graph of the internal angles of rectangle triangles with cathetus no larger than 30, 50, 80 or 120 units (as in Figure 18).

Figure 21: Perspective view of the first, second and third kind of bar.

Description of the invention

The idea for this invention came from a small personal study of the Pythagorean Theorem that revealed the infinite number of rectangle triangles with integer sides. Most small rectangle triangles with integer sides either belong to one of two series of equations or are defined by a multiple of one such equation. The two series of equations are

$$(3+2i)^2+[2(i+1)(i+2)]^2=[1+2(i+1)(i+2)]^2$$
$$(4+4i)^2+[(2i+1)(2i+3)]^2=[2+(2i+1)(2i+3)]^2$$

where "i" is a non-negative integer. For i=0, the two series define the same rectangle triangle ($3^2+4^2=5^2$). Both series define a rectangle triangle $\boldsymbol{A}^2+\boldsymbol{B}^2=\boldsymbol{D}^2$ where $\boldsymbol{D}=\boldsymbol{B}+1$ or $\boldsymbol{D}=\boldsymbol{B}+2$. The three sides of a rectangle triangle may be multiplied by another integer "**n**" resulting $(\boldsymbol{n}\mathrm{x}\boldsymbol{A})^2+(\boldsymbol{n}\mathrm{x}\boldsymbol{B})^2=(\boldsymbol{n}\mathrm{x}\boldsymbol{D})^2$. One example of integer rectangle triangle that does not belong into this set is $20^2+21^2=29^2$.

An exhaustive investigation was conducted that showed the existence of 83 integer rectangle triangles with both cathetuses not longer than 120 (see Figure 18). These rectangle triangles provide a reasonable coverage of angles (see Figure 20).

This small study unchained a desire for wooden furniture using the Pythagorean Theorem but none was available commercially. A contacted carpenter agreed to make a set of square section bars because he had a machine that could drill a large number of parallel holes simultaneously. Thus, this machine defined the actual unit length: 96mm. Initial tests demonstrated that this unit length allowed at most 6 different integer rectangle triangles to be used inside a typically sized home (the total length of the bars should not exceed 20 of these units). Quite luckily, a special drilling pattern was devised that allowed the use of the same machine and allowed 66 different integer rectangle triangles to be used inside home. This pattern is defined in claim 1.

Now the problem was that all triangles were either parallel or perpendicular to each other (because the bar's section is square). The solution is, then, to use non-square sections. The curious thing about this solution is that there is only another even-sided regular polygon whose angles have rational cosines: the hexagon.

Again, this caused a problem. What must be the size of this hexagon so that one may assemble triple connections containing both square section bars and hexagonal section bars? The solution is defined in claim 1. The hexagonal bars of this size and with a drilling pattern similar to the square bars are the second kind of bars. So, now, what must be the drilling pattern so that one may assemble triple connections containing only hexagonal section bars? That is defined in claim 1. These solutions leave us with two different kinds of drilling patterns for bars with the same hexagonal section and, therefore, leave us without triple connections containing: (i) two bars of first kind and one bar of third kind; (ii) two bars of third kind and one bar of first kind; (iii) two bars of second kind and one bar of third kind; (iv) two bars of third kind and one bar of second kind; (v) bars of all three kinds; (vi) only bars of second kind.

This remaining problem seems, at present, unsolvable without unnecessary complications. Furthermore, the three kinds of bars are somewhat mutually compatible in other ways. "Somewhat" compatible because integer multiples or roots of irrational quantities may be very, very close to integer quantities. As an example, consider angles whose cosines have absolute values of one half and one third. The sines of these angles are $\sqrt{3}/2$ and $(2/3)\times\sqrt{2}$ respectively. So, even though these sines are irrational, one should note that

$$15\times\sqrt{3}/2\approx13 \text{ and } 105\times(2/3)\times\sqrt{2}\approx99.$$

Another example is that one may build one triangle with integer sides of two different unit lengths (whose ratio is irrational) and still have a rational cosine (of course, this is not exactly true but very good approximations may be found). This is the reason why the third kind of bar are "somewhat" compatible with the other kinds in respect to the assembly of triangles. It is always possible to make triangles with sides of any length (as far as no length is greater than the sum of the other two) but it is not always possible to make triangles with given length ratios and given angles. Nevertheless, the drilling patterns of second and of third kind of bars are defined in claim 1 to impose a mutual unit length ***G*** that avoids much interference with the other unit length of the second kind of bars because $\boldsymbol{G}=(10/3)\times\boldsymbol{J}=(10/3)\times\sqrt{6}\times(\sqrt{3}-1)\times\boldsymbol{L}\approx6\times\boldsymbol{L}$.

Detailed description of the invention

The basic elements of the present invention can produce, among other sub-structures, the ones named "triple connections" and "quadruple connections". These are described below.

A triple connection is a connection of three bars where no two bars are parallel and where each pair of bars is connected at a single point (there are three connection points in a triple connection). Triple connections are very significant in this invention because they implement a simple and mechanically robust way to move beyond planar structures and into the third dimension. The three bars kit provides four kinds of triple connections (see Figures 4, 5, 6 and 7). Any pair of bars in a triple connection defines a plan and the third bar of that same triple connection is both out of that plan and fixed at two points.

A quadruple connection is a group of four bars mutually connected at four points, two of the bars being of the same kind and the other two bars being of some other kind and parallel. There is a very large number of different quadruple connections but one may be set apart. This special quadruple connection is made of two parallel bars of first kind and two bars of second kind. When in the special configuration, the absolute value of the cosine of the angle between two bars of different kind is one half (see Figure 16). No other quadruple connection with rational cosines for the angles between a parallel bar and an oblique bar was found. The importance of rational cosines is described below. Also worth mentioning is another quadruple connection made with two bars of third kind and two parallel bars both of second and third kind. It is possible to assemble this connection so that the angles between any of the two oblique bars and any of the two parallel bars are exactly 45 degrees (see Figure 17).

Structures containing triangles whose edges have integer length may be designed considering the law of cosines. Triangles are the only kind of planar polygons whose internal angles are inherently fixed and, therefore, triangles are required for rigid structures.

Supposing a triangle with sides of length ***A***, ***B*** and ***D***, the law of cosines states that

$$D^2=A^2+B^2+2\times A\times B\times C$$

where ***C*** is the cosine of the supplement of the angle between sides ***A*** and ***B***. If ***A*** and ***B*** are integers then ***D*** cannot be integer unless ***C*** is a rational number. The main idea of this invention is that ***C*** is a rational number for all the angles between any two bars in any triple connection. The absolute value of ***C*** is one third for the triple connection of Figure 7 and is one half or null for the triple connections of Figures 4, 5 and 6. The important consequence of this setup is that there is a large number of ways to make and reinforce generic three-dimensional structures.

The graphs in Figures 18 and 19 show values of ***A***, ***B*** and ***C*** for which ***D*** is integer (***C*** being a rational number). Only the five values of ***C*** relevant for triple connections are used in these graphs.

It should be noted that connections between bars are supposed to be made through the holes with standard industrial connectors like rivets, nuts and bolts or the like. It should also be noted that the law of cosines is still valid when ***A***, ***B*** and/or ***D*** are not integer and/or ***C*** is not rational.

The first kind of bar has uniform square convex hull section along its length. The second and third kind of bars, have equal and uniform regular hexagonal convex hull sections along their lengths. The height ***H*** of the hexagonal convex hull section of the second and third kind of bars is related to the length ***L*** of the side of the square convex hull section of the first kind of bars by the following formula: $\boldsymbol{H}=(\sqrt{3}-1)\mathrm{x}\boldsymbol{L}$.

The three bars (of first, second or third kind) have holes directed perpendicularly to the sides of the convex hulls, directed perpendicularly to the length of the bars and crossing exactly through the centre of the convex hulls, all holes on each bar having the same diameter.

The first kind of bar has holes distributed periodically along its length, with a single wave vector of norm 2x***L***, one hole every ***L***/2 length and changing sides only once every ***L*** length.

The second kind of bar has holes distributed periodically along the length of the bar, with two different wave vectors, one wave vector having norm 2x***L*** and the other wave vector having norm ***G***/2=5x***J***/3, where $\boldsymbol{J}=\sqrt{6}\boldsymbol{H}$.

The second kind of bar also has holes on two pairs of opposite sides distributed periodically along the length of the bar, one hole every ***L***/2 length and changing to the other of the two pairs of opposite sides only once every ***L*** length.

The second kind of bar additionally has holes on one pair of opposite sides distributed periodically along the length of the bar, one hole every ***G***/2 length.

Finally, the second kind of bar can also have one reference hole on the ***G***/2 wave vector side(s) such that its geometric centre is exactly at mid-distance between two adjacent pairs of holes on the other side(s).

The third kind of bar has holes on all of its sides, distributed periodically along its length, with a single wave vector, with one hole every ***J****/3* or ***J***/6 length, holes separated by a distance ***J*** being on different sides and existing at least one chosen side where holes separated by a distance 2x***J***/3 are on that same chosen side.

Any bar (of first, second or third kind) cooperates with any other bar (of first, second or third kind) by means of some hole, configuring a rotation axle without angular limits.

Any bar (of first, second or third kind) can also cooperate with any other bar (of first, second or third kind) with wave vectors and sides scaled by a factor 1/4, 1/3, 1, 3, or 4, by means of two or more holes, configuring a reinforced or extended composite bar, with the exception of the first and third kind of bars which cannot mutually cooperate by means of more than one hole.

The height $\boldsymbol{H}$ of the second and third kind of bars must be $(\sqrt{3}-1)\mathrm{x}\boldsymbol{L}$, where $\boldsymbol{L}$ is the length of the side of the first kind of bars, for both triple connections of Figures 5 and 6 to be possible.

All bars must have holes perpendicular to their convex hull surface so that standard industrial connectors may be used. Even distribution of stresses is ensured when the holes cross the centre of the bar's section.

The first and second kind of bars have holes distributed periodically along their lengths, with a wave vector of norm $2\mathrm{x}\boldsymbol{L}$, one hole every $\boldsymbol{L}/2$ length and changing side only once every $\boldsymbol{L}$ length, because of two reasons: (i) a reduced unit length ($\boldsymbol{L}/2$) increases the angle coverage for the same maximum bar length and (ii) only holes on different sides and separated by a distance $\boldsymbol{L}$ allow the triple connections of Figures 4, 5 and 6.

The second and third kind of bars are easier to use together if they share at least one wave vector on at least one side. The wave vector of norm $\boldsymbol{G}/2=(5/3)\mathrm{x}\boldsymbol{J}$, where $\boldsymbol{J}=\sqrt{6}\mathrm{x}\boldsymbol{H}$, was chosen because its holes most unlikely intersect holes of the other wave vector of second kind of bars. When holes intersect, the rigidity of the bars is impaired. Nevertheless, given that $\boldsymbol{G}$ is not exactly $6\mathrm{x}\boldsymbol{L}$, intersections may occur and only one hole on a $\boldsymbol{G}/2$ wave vector side can be placed rigorously. This is the "reference hole". Only if it is placed at mid-distance between two adjacent holes on other side(s) can it avoid the verification of the direction of bars when assembling bars in parallel (see Figure 10).

The third kind of bars can be assembled in a triple connection (as in Figure 7) only if they have holes on different sides separated by a distance ***J***. Angle coverage is, again, increased if one uses shorter unit lengths. A reasonable unit length is ***J***/3 because it represents not too many holes and implies drilling patterns recognisably different from those of the second kind of bars.

Enhanced flexibility of structural design results from the possibility of total angular freedom for single connector connections and from the possibility of using bars in different scales.

Preferred Embodiment

The structures may contain instances of just one kind of bar, instances of any pair of kinds of bar or instances of all three kinds of bar.

The first and second kind of bars may have any length above ***L*** and third kind of bar may have any length above 2x***J***/3, but we recommend, for the maximum distance between parallel holes in a given bar, the following sequence of multipliers of ***U***=2x***L*** for the first and second kind of bars, or multipliers of ***V***=2x***J***/3 for the third kind of bar:

3, 5, 7, 10, 13, 17, 21, 26 and 31.

Supposing this sequence is used, each kind of bar exists with nine different lengths. Supposing, also, that ***L*** is equal to 32mm, the longest first and second kind of bars have a total length of 2016mm.

Furthermore, each kind of bar may exist in several different scales. For any given reference side of length ***L***, a given structure may contain both bars of side length equal to ***L*** and bars of side length equal to ***K***x***L*** where ***K*** may be 1/4, 1/3, 3, 4 or products of integer powers of these values. We recommend the use of just two values of ***K***, one value being the reciprocal of the other value.

Supposing that each bar exists in three different scales in addition to existing with nine different lengths, there is a total of eighty-one different instances of bars.

It is very convenient to mark each hole with a number corresponding to its distance from the first hole on the same side of the bar. Also, reference holes of the second kind of bars should be clearly marked. The definition of "reference hole" is given in claim 2 but one should note that only the second kind of bars have reference holes and that each bar may have at most one reference hole.

The bars may be manufactured by means of different processes and using many different materials but, due to the fact that there are only three kinds of bars, we believe that the process of injection can be optimized efficiently for many different instances of bars. It should be noted, however, that different instances may be manufactured by different processes. Furthermore, given that most standard connectors will induce compression stresses along the holes, it is foreseen as adequate both the avoidance of intersections between holes and the full extension of their sleeves from side to side of the convex hulls.

It must be clear that the present three bars kit, described before, is simply one possible example of implementation, merely stating clearly the principles of the invention. Variations and modifications to the cited kit can be made without moving away from the scope and principles of the invention. All these modifications and variations must be enclosed in the scope of the present invention and be protected by the following claims.

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CLAIMS

1. Three bars kit to build polyhedric structures based on the law of cosines, where one of the bars, the bar of the first kind, has uniform square convex hull section along its length, both the other two bars, the bars of the second and third kinds, have equal and uniform regular hexagonal convex hull sections along their lengths and all kinds of bars have holes directed perpendicularly to the sides of the convex hulls, directed perpendicularly to the length of the bars and crossing exactly through the centre of the convex hulls, wherein:

the height $\boldsymbol{H}$ of the hexagonal convex hull section is related to the length $\boldsymbol{L}$ of the side of the square convex hull section by the following formula: $\boldsymbol{H}=(\sqrt{3}-1)\mathrm{x}\boldsymbol{L}$,

the first kind of bar has holes distributed periodically along its length, with a single wave vector of norm $2\mathrm{x}\boldsymbol{L}$, one hole every $\boldsymbol{L}/2$ length and changing sides only once every $\boldsymbol{L}$ length,

the second kind of bar has holes distributed periodically along the length of the bar, with two different wave vectors, one wave vector having norm $2\mathrm{x}\boldsymbol{L}$ and the other wave vector having norm $\boldsymbol{G}/2=5\mathrm{x}\boldsymbol{J}/3$, where $\boldsymbol{J}=\sqrt{6}\boldsymbol{H}$,

the second kind of bar has holes on two pairs of opposite sides distributed periodically along the length of the bar, one hole every $\boldsymbol{L}/2$ length and changing to the other of the two pairs of opposite sides only once every $\boldsymbol{L}$ length,

the second kind of bar has holes on the remaining pair of opposite sides distributed periodically along the length of the bar, with one hole every $\boldsymbol{G}/2$ length, the third kind of bar has holes on all of its sides, distributed periodically along its length, with a single wave vector, with one hole every ***J***/3 or ***J***/6 length, holes separated by a distance ***J*** being on different sides and existing at least one chosen pair of opposite sides where holes separated by a distance 2x***J***/3 are on that same chosen pair of opposite sides.

2. Three bars kit, according to claim 1, wherein the second kind of bar has one reference hole on the ***G***/2 wave vector side(s) such that its geometric centre is exactly at mid-distance between two adjacent holes on the other side(s).

3. Three bars kit, according to claims 1 and 2, wherein any kind of bar cooperates with any other kind of bar by means of some hole, configuring a rotation axle without angular limits.

4. Three bars kit, according to claims 1, 2, and 3, wherein any kind of bar cooperates with any other bar with wave vectors and sides scaled by a factor 1/4, 1/3, 1, 3, or 4, by means of two or more holes, configuring a reinforced or extended composite bar, with the exception of bars the first and third kinds which cannot mutually cooperate by means of more than one hole.

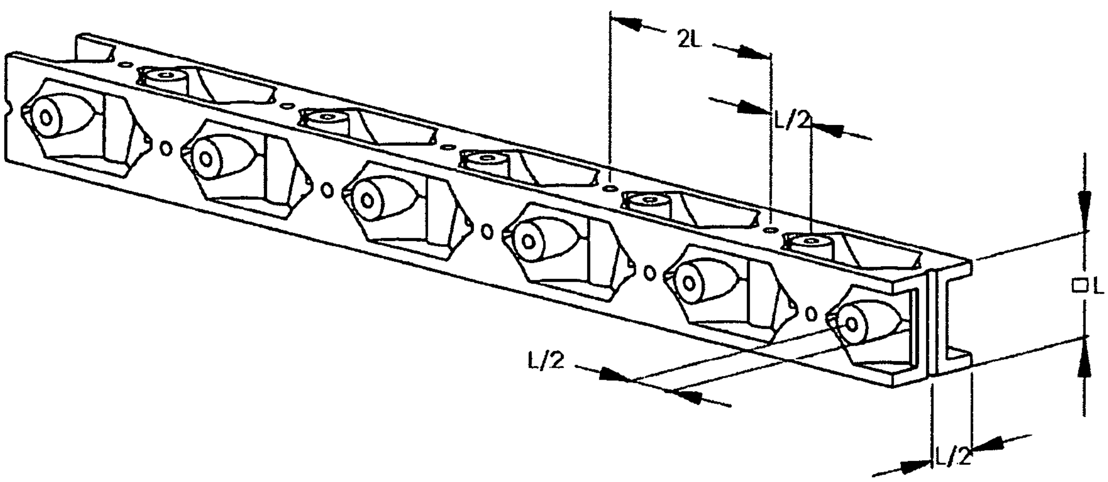

Figure 1

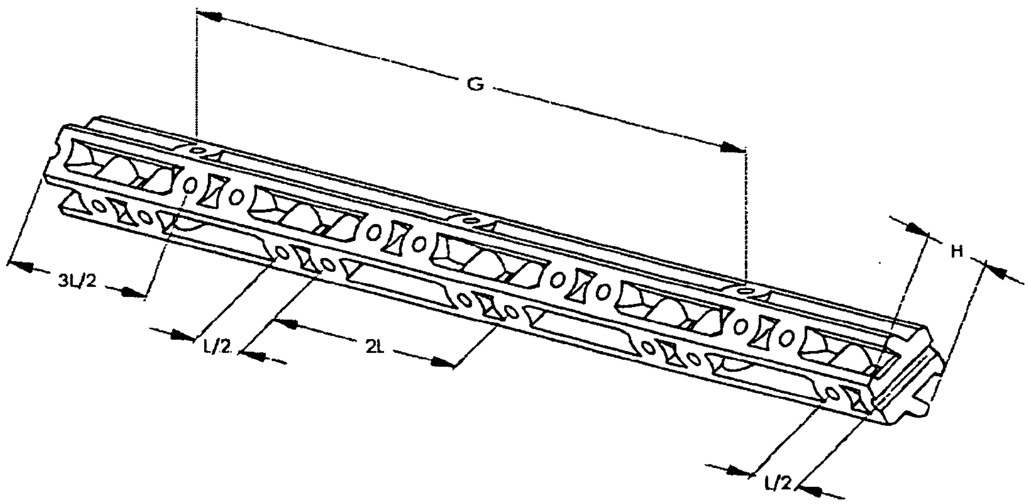

Figure 2

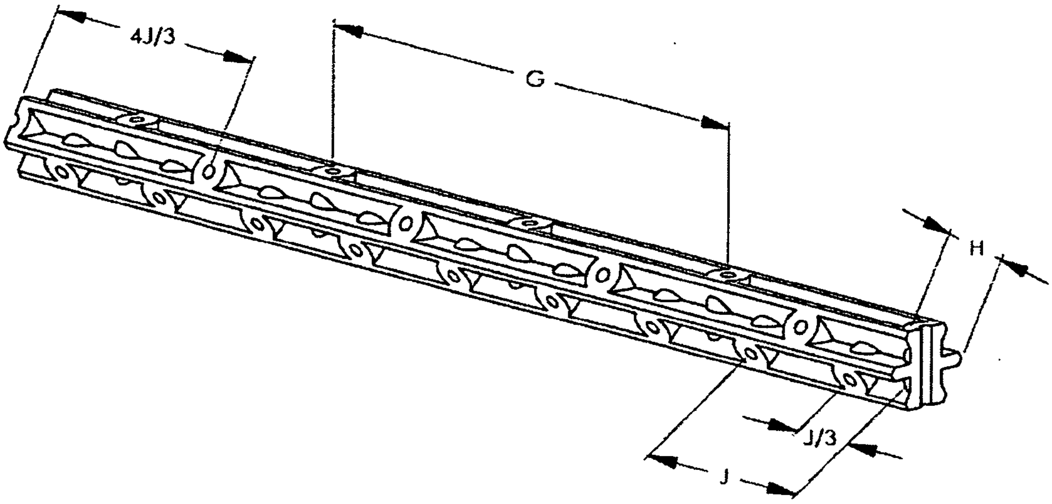

Figure 3

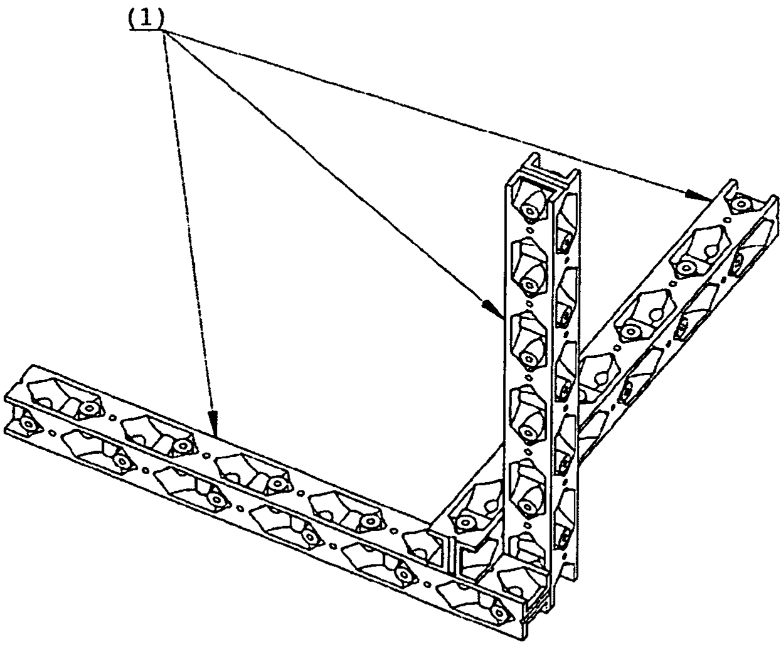

Figure 4

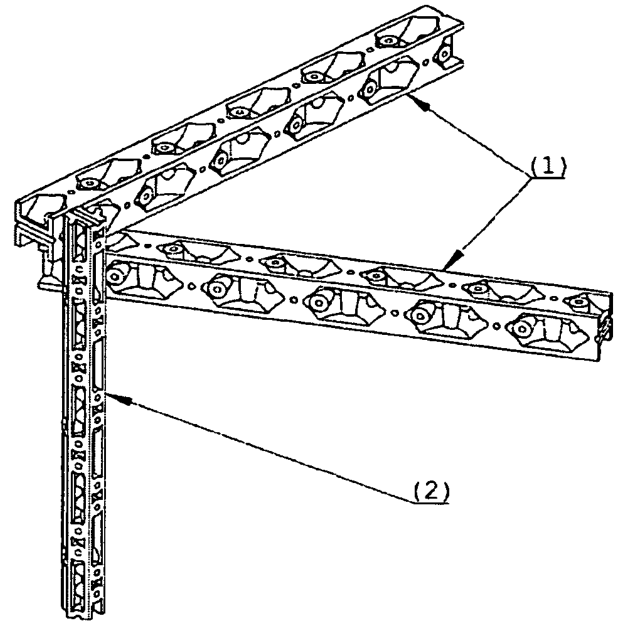

Figure 5

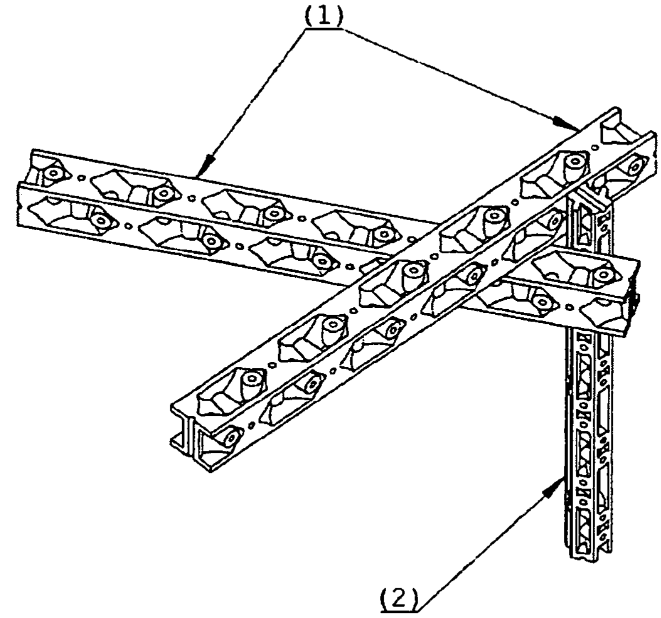

Figure 6

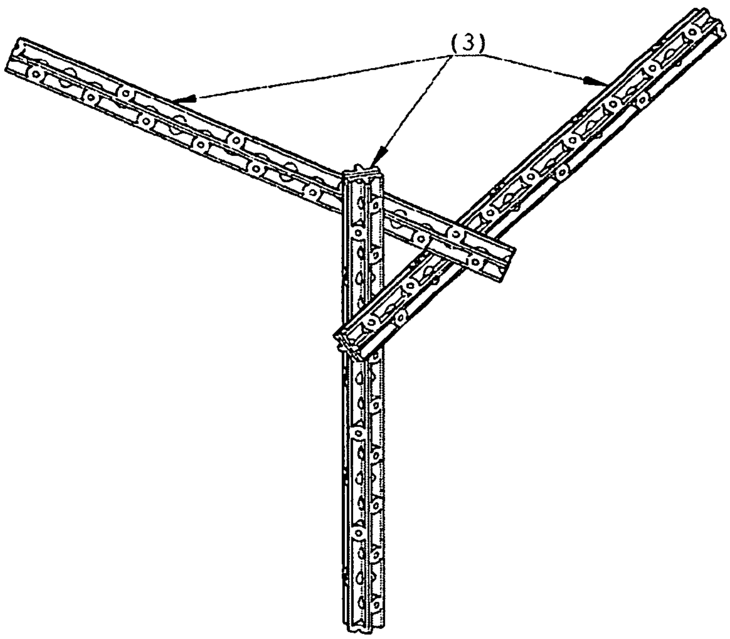

Figure 7

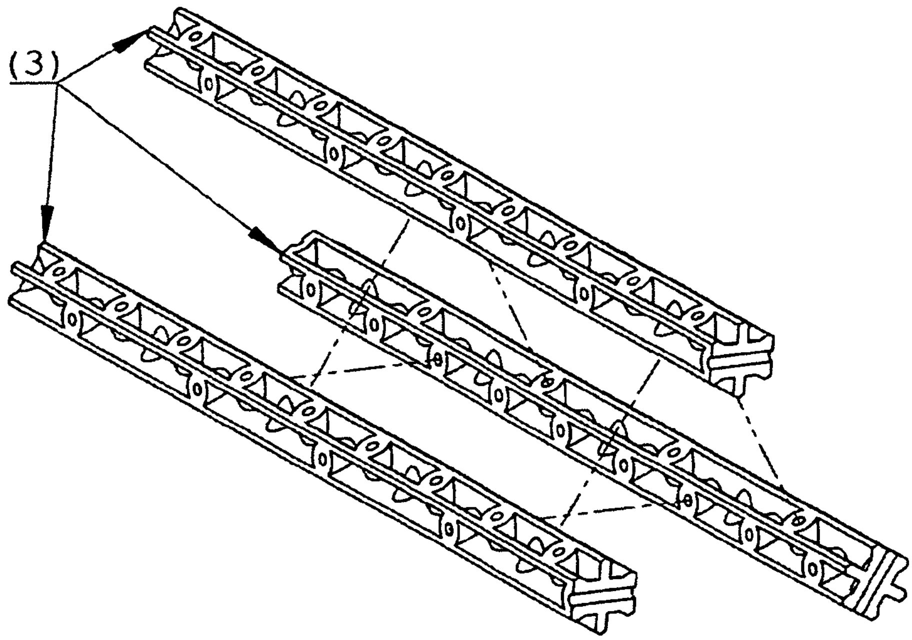

Figure 8

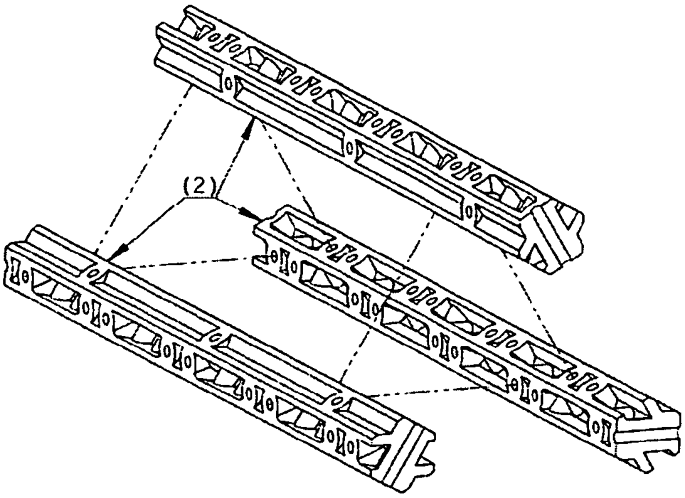

Figure 9

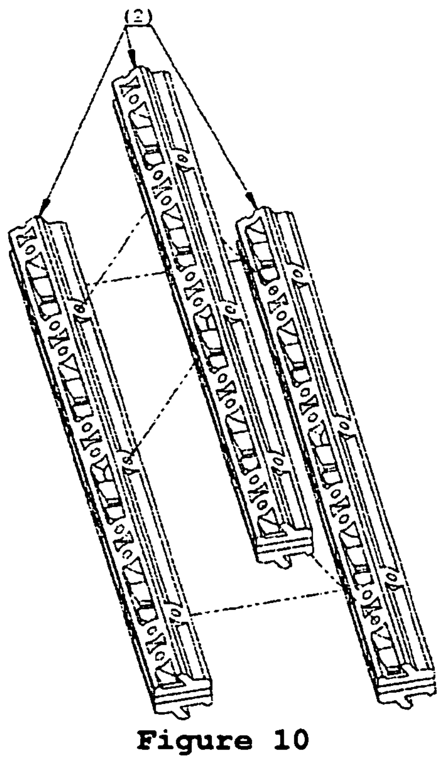

Figure 10

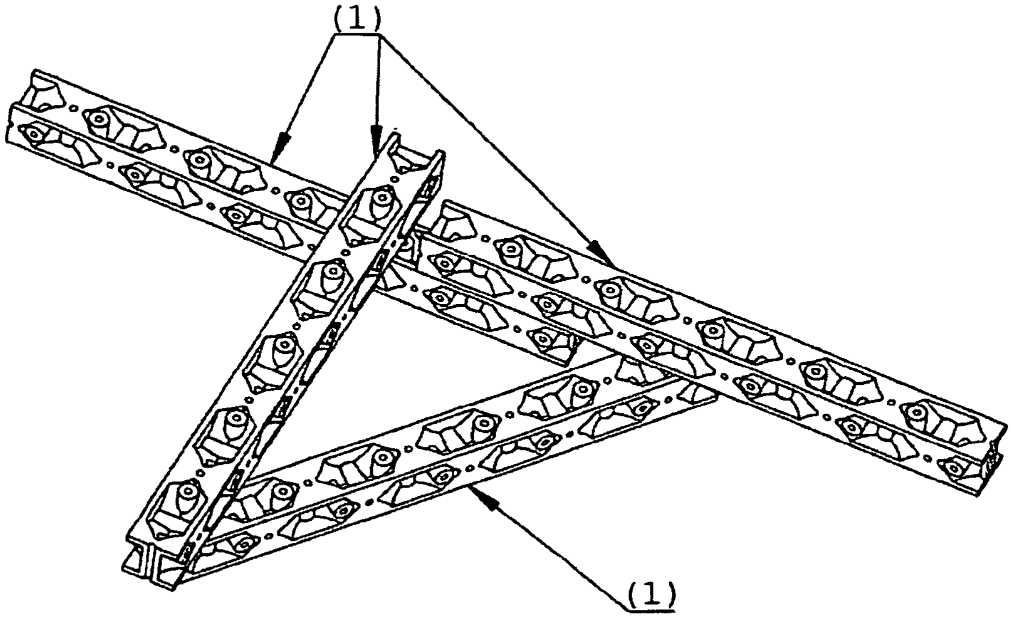

Figure 11

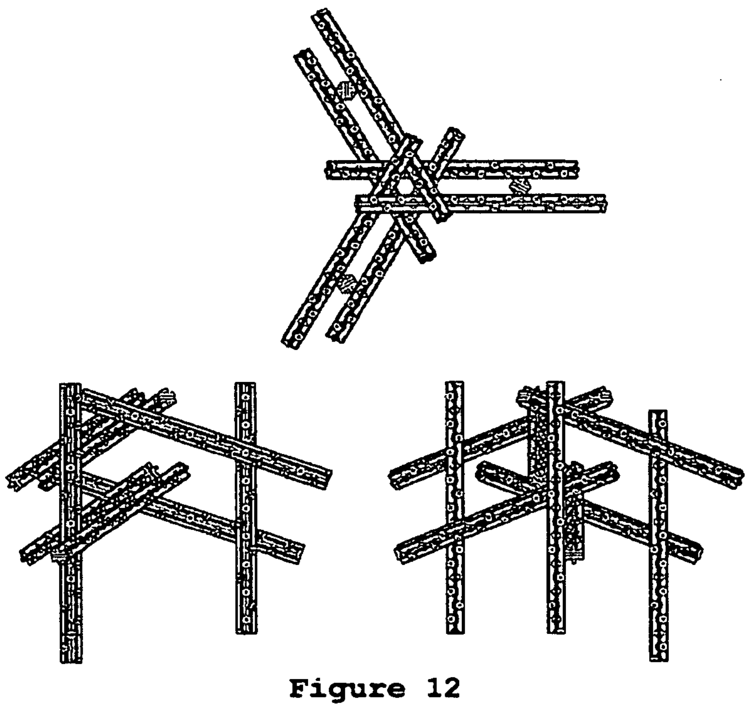

Figure 12

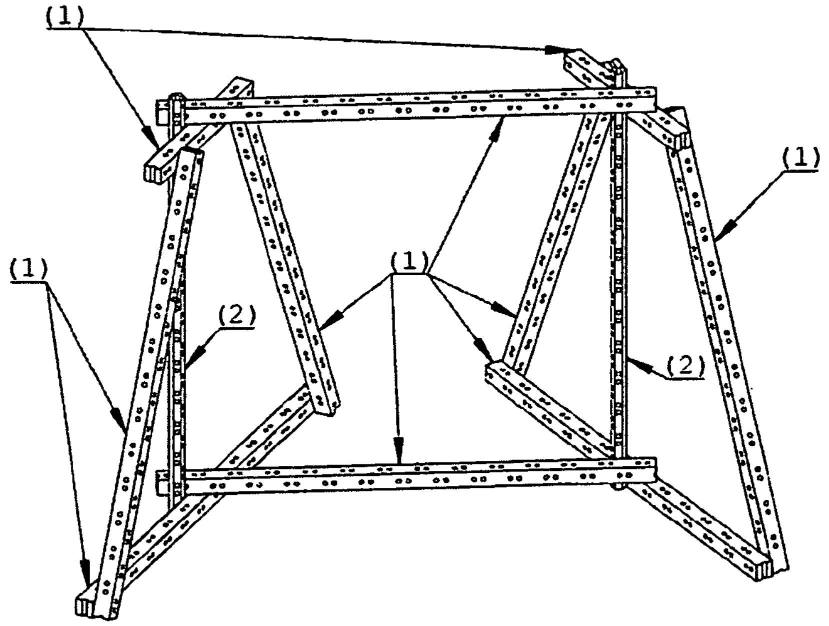

Figure 13

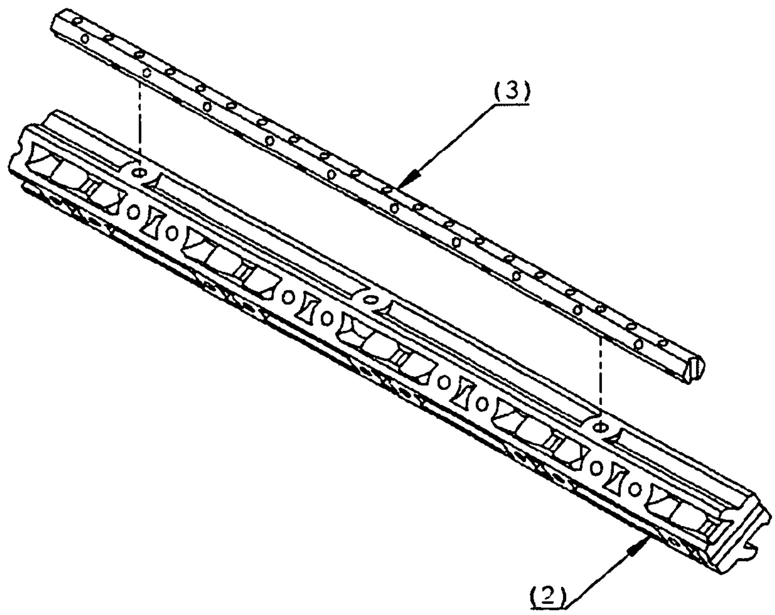

Figure 14

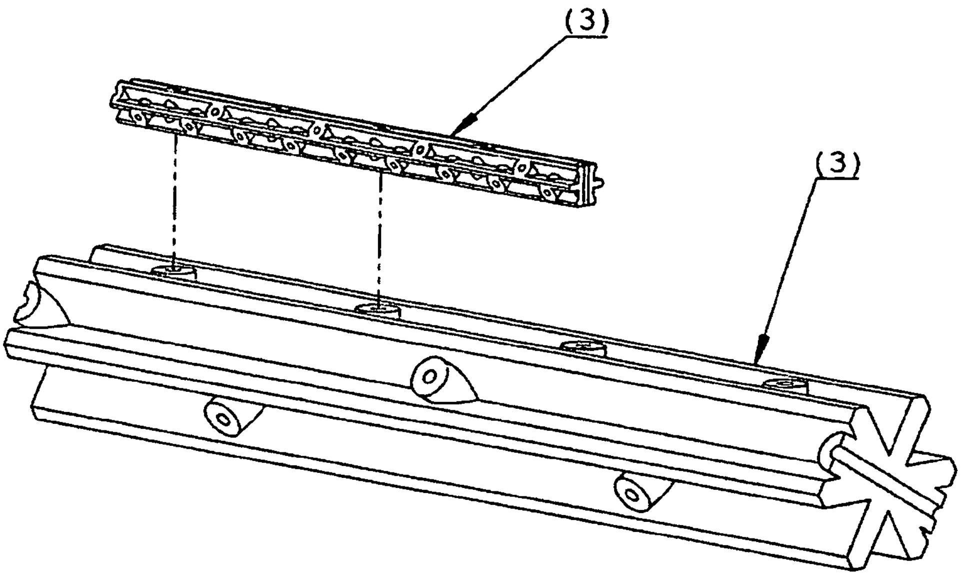

Figure 15

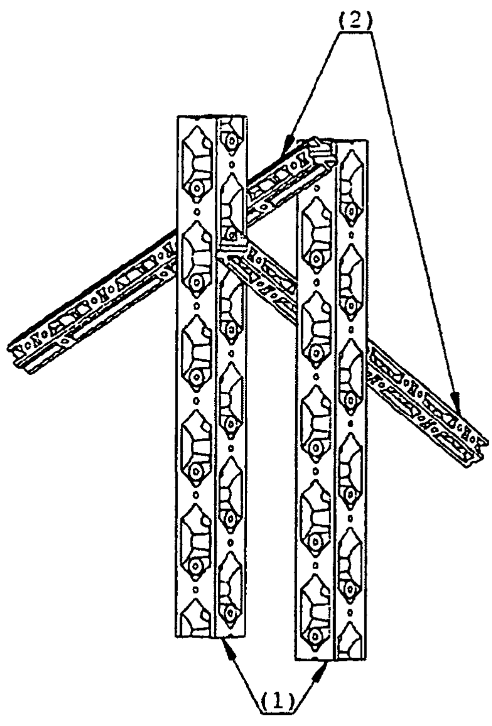

Figure 16

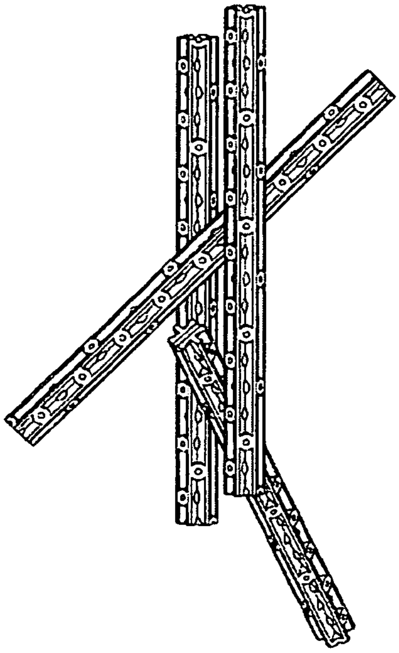

Figure 17

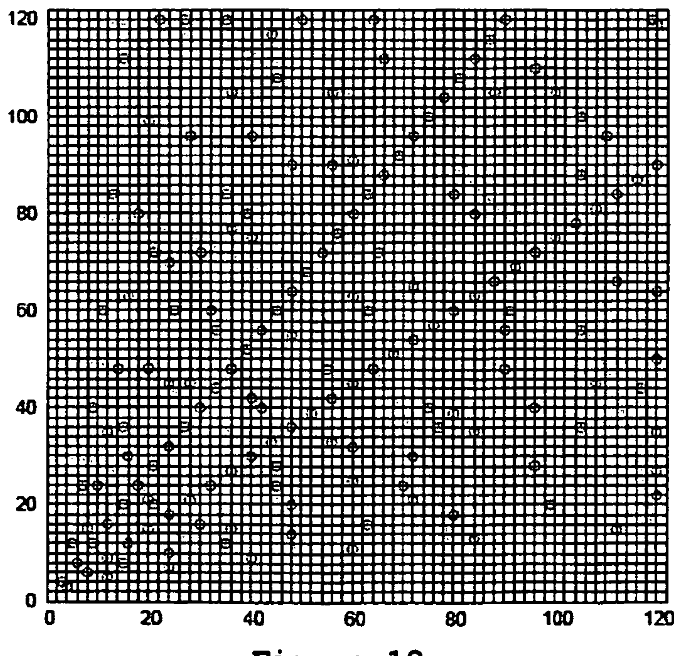

Figure 18

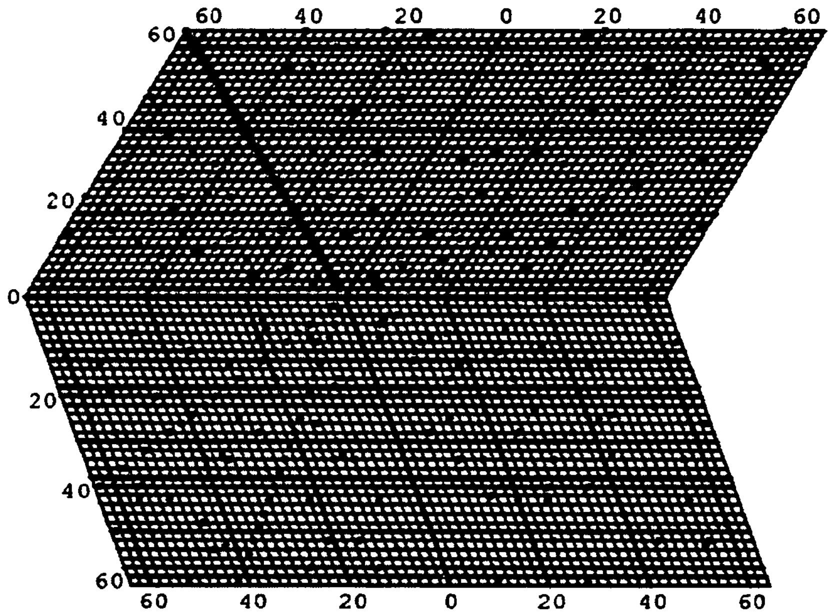

Figure 19

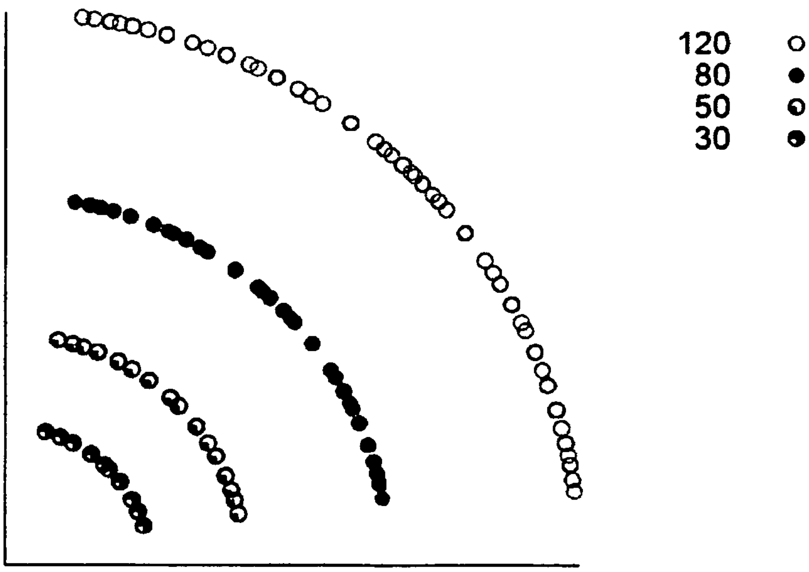

Figure 20

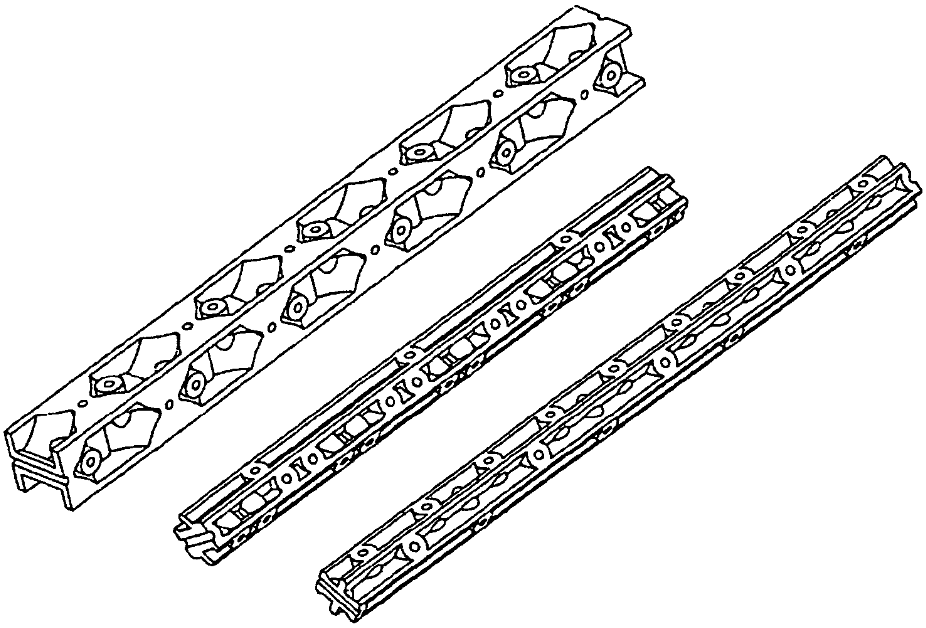

Figure 21