What is the smallest number of circles that can cover more area of a square than a single circle? An example of the scientific method in action.

Luís Nobre Gonçalves

http://matagalatlante.org/UiTlecture.pdf

Abstract

The financial crisis is ravaging the economy and opinion–makers are calling for innovation as a solution. Many have forgotten that innovation is nothing but a side–effect of the old, well–established, classic scientific method.
This lecture is an attempt to remember what the scientific method is and how it works. A problem related with circle packing is used as a demonstrative example. Some applications in Materials Science are explored.

Outline

1. Introduction

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2. Answer

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3. Go beyond

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4. Summary

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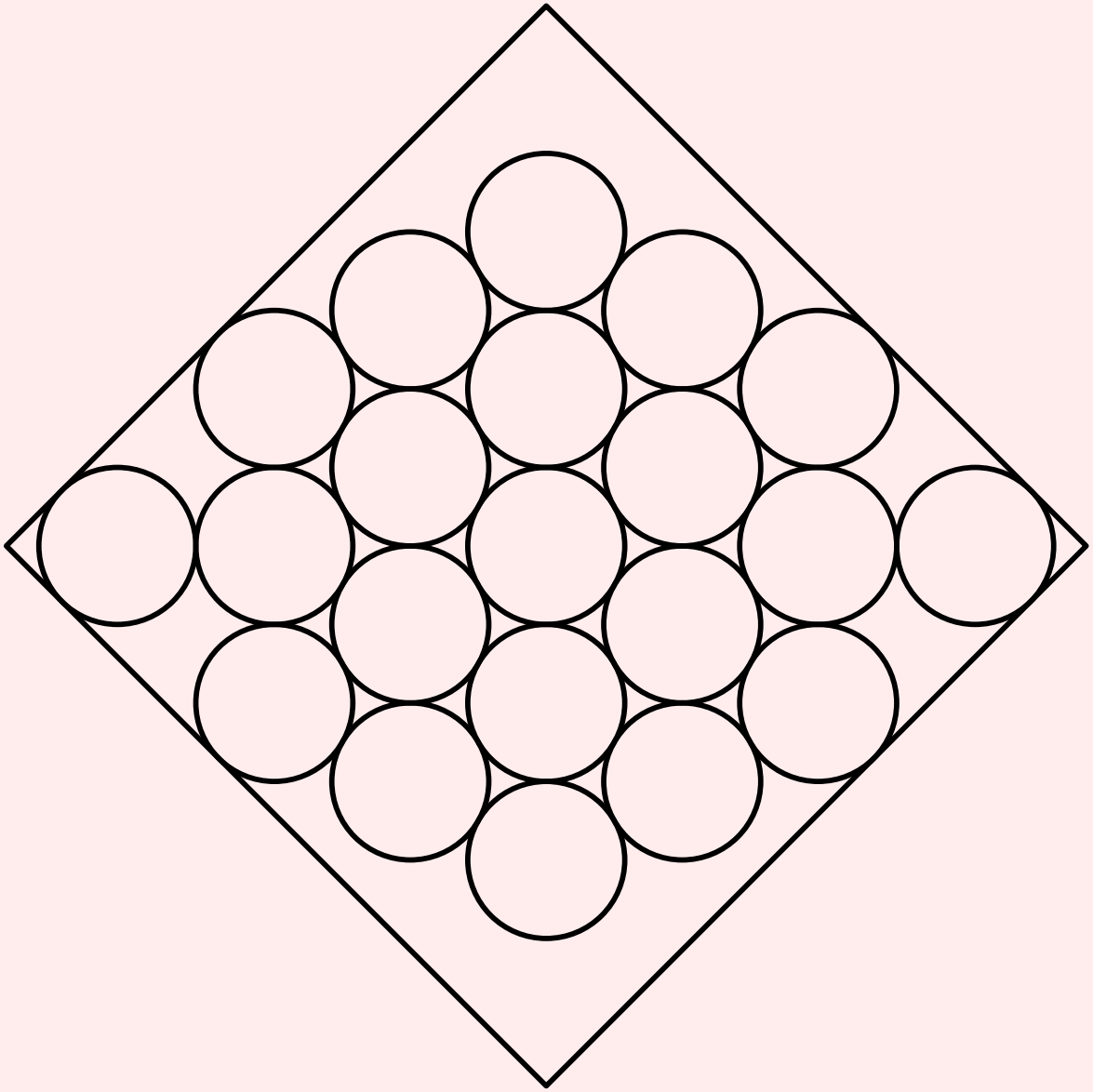
5. References

Introduction



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Faculdade de Ciências e Tecnologia

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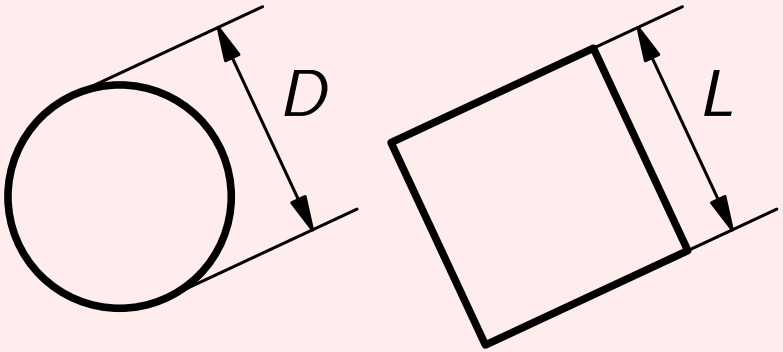
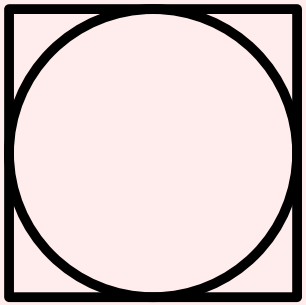
Definitions



$$F = \frac{NA_{\bigcirc}}{A_{\square}} = \frac{N\pi\left(\frac{D}{2}\right)^2}{L^2(N,D)}$$

N is the the number of circles

Target fraction of covered area (0,785)



$$N = 1$$

$$L = D$$

$$F_T = \frac{\pi}{4} \approx 78{,}5\%$$

The Scientific Method (SM)

- To learn from reality

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  - Avoid belief, bias and myth

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  - Everybody learns

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- Sophisticated trial–and–error
  - Everybody learns
  - Models

The Scientific Method (SM)

- To learn from reality
  - Avoid belief, bias and myth
- Sophisticated trial–and–error
  - Everybody learns
  - Models should be simple, general and beautiful

Models

Models are

- mental representations
- visualizations of uninitiated actions
- expectations about the unknown
- scenarios
- expressions

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- ways to start, continue, assume, measure, facilitate, avoid common problems, interpret, classify, detect, solve, confirm, prove, explain, generalize, understand, use, teach and publish

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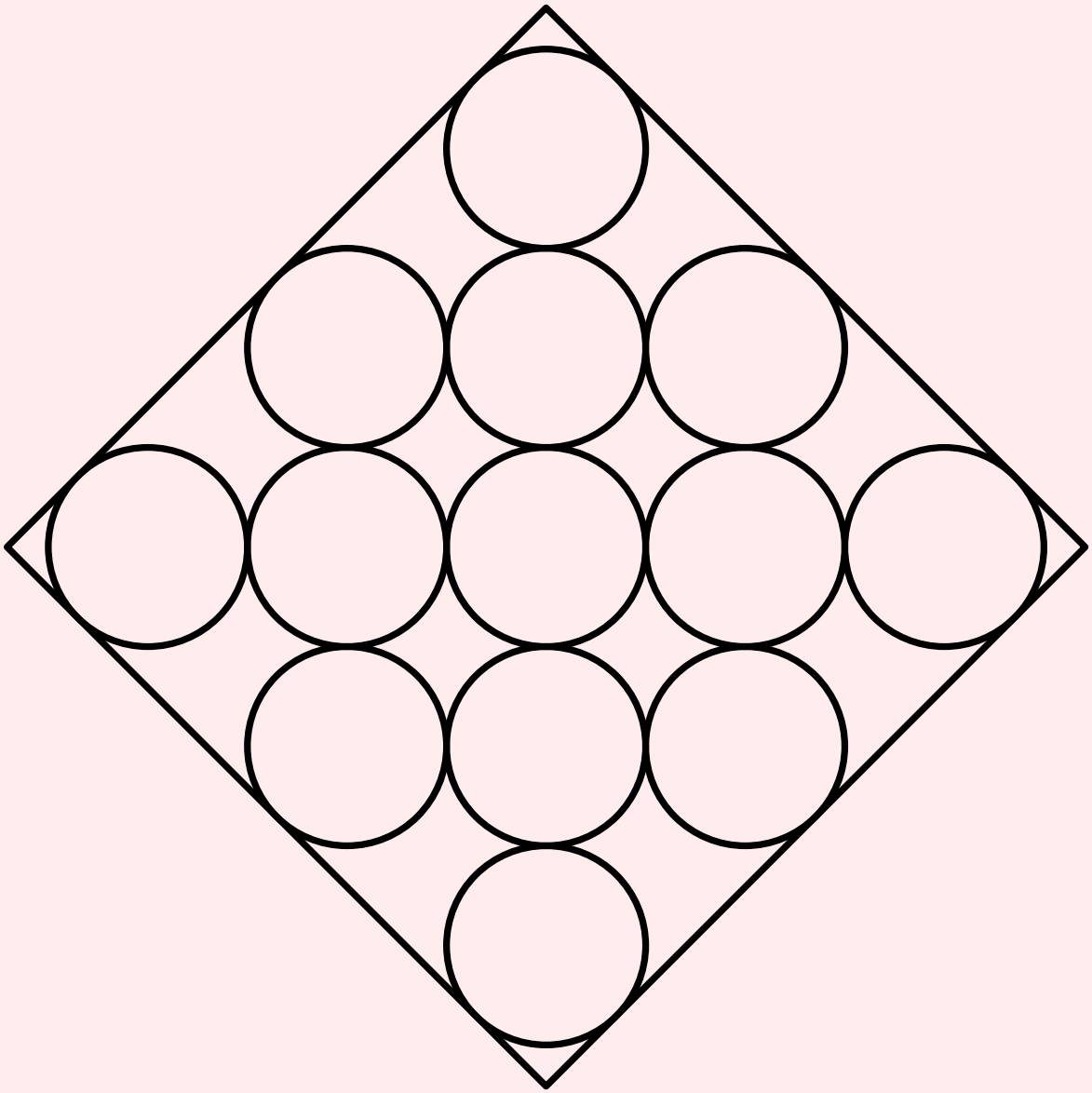
Models are

- mental representations
- visualizations of uninitiated actions
- expectations about the unknown
- scenarios
- expressions
- ways to start, continue, assume, measure, facilitate, avoid common problems, interpret, classify, detect, solve, confirm, prove, explain, generalize, understand, use, teach and publish

A scientist works to improve not just one model but several.

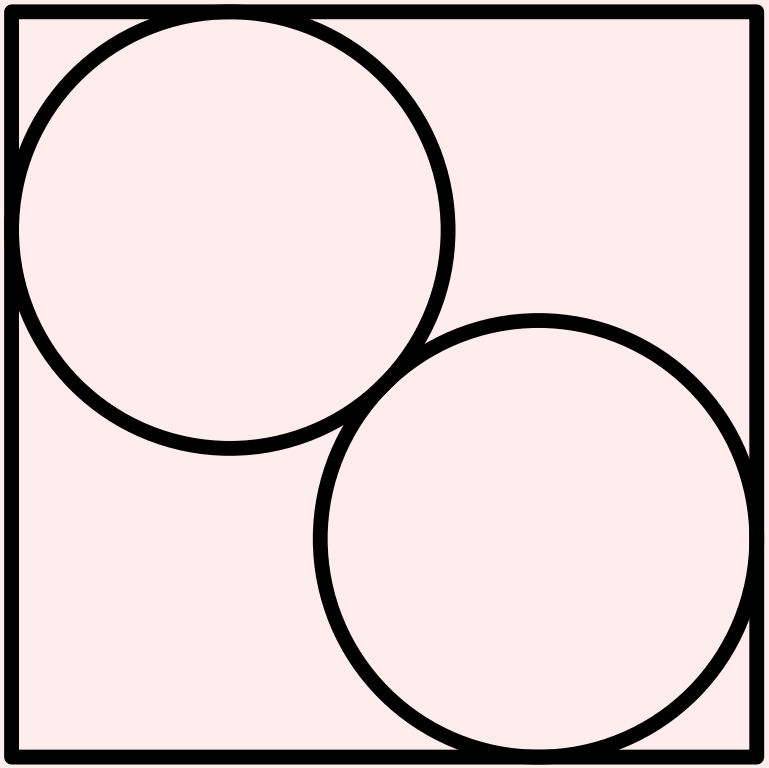
Answer



(SM)

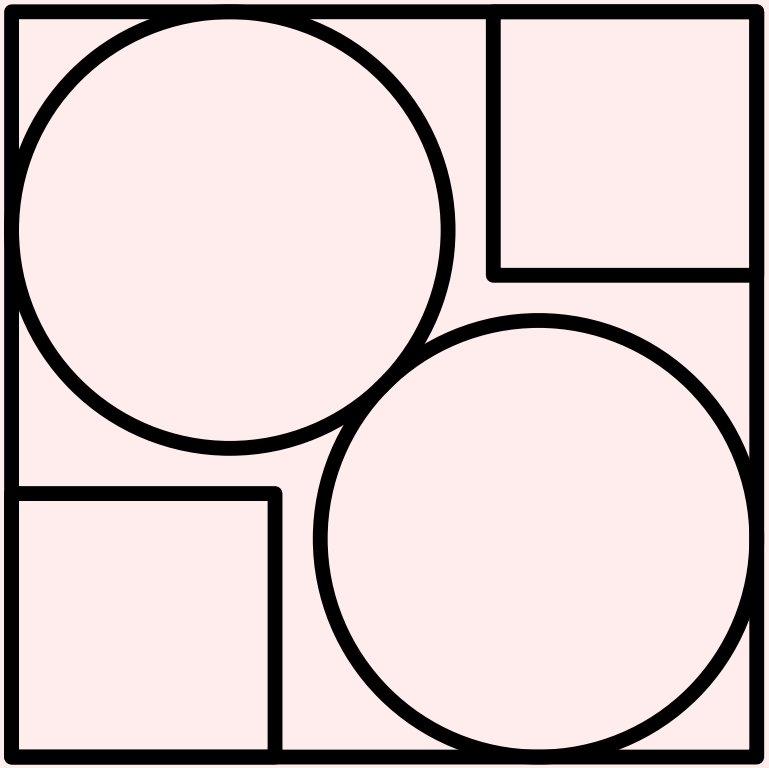
- Start at the beginning
- What's the best way to reach the answer?

2



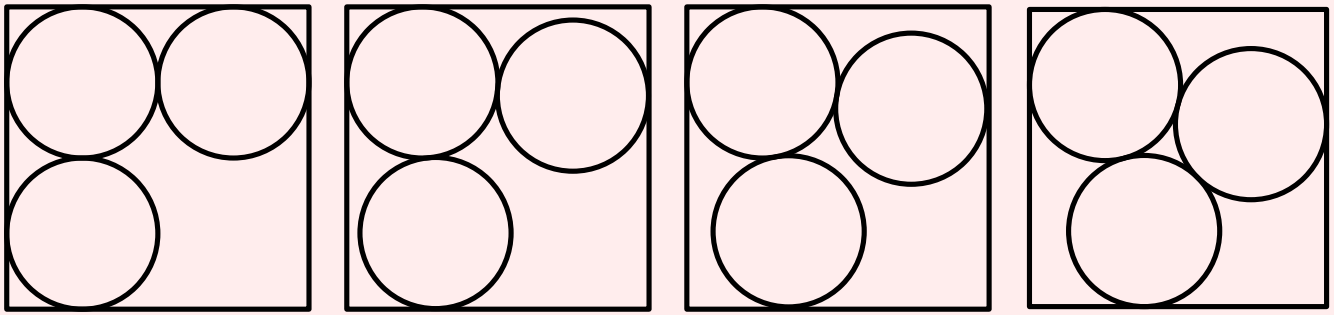
$$F_2 \approx 0{,}539$$

2



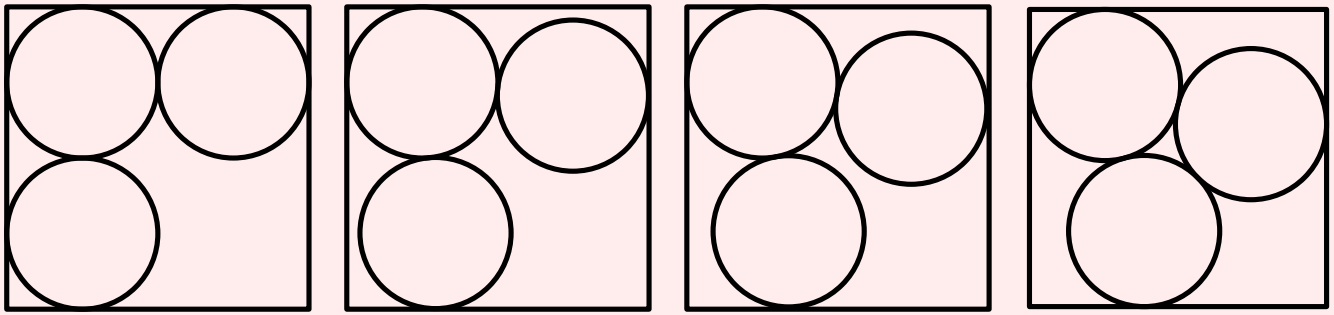
$$F_2 \approx F_T - 25\%$$

3, 5

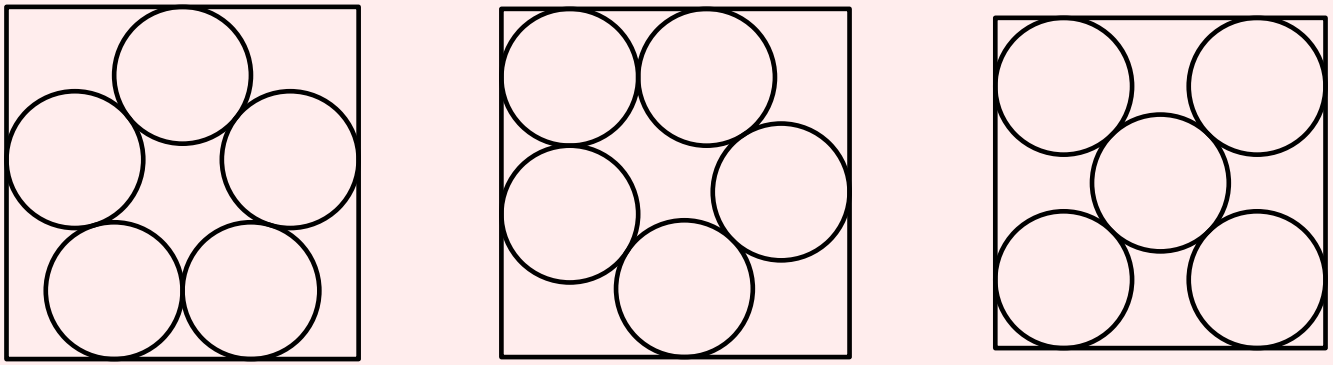


$$F_3 \approx 0{,}610$$

3, 5



$$F_3 \approx 0{,}610$$

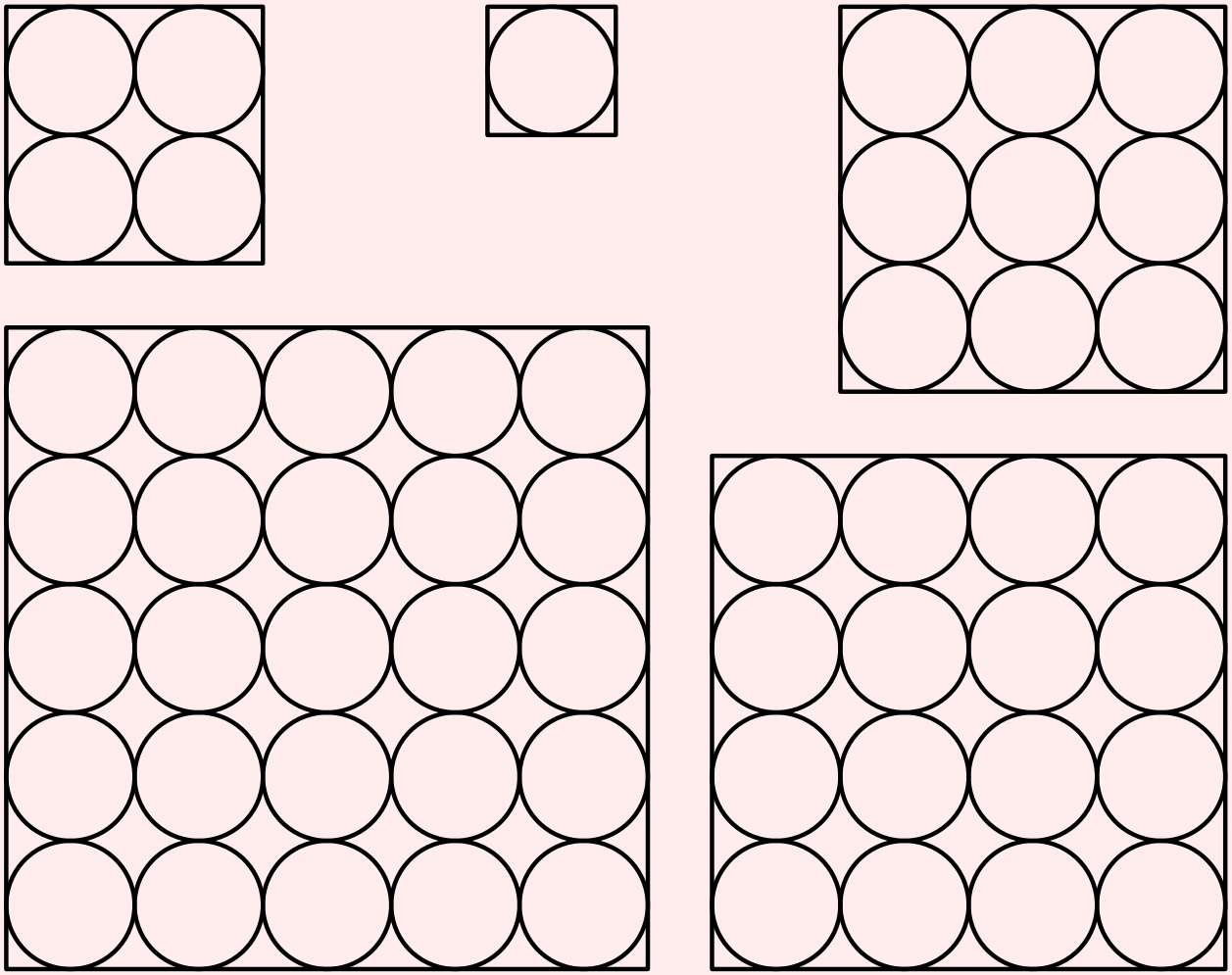


$$F_5 \approx 0{,}674$$

(SM)

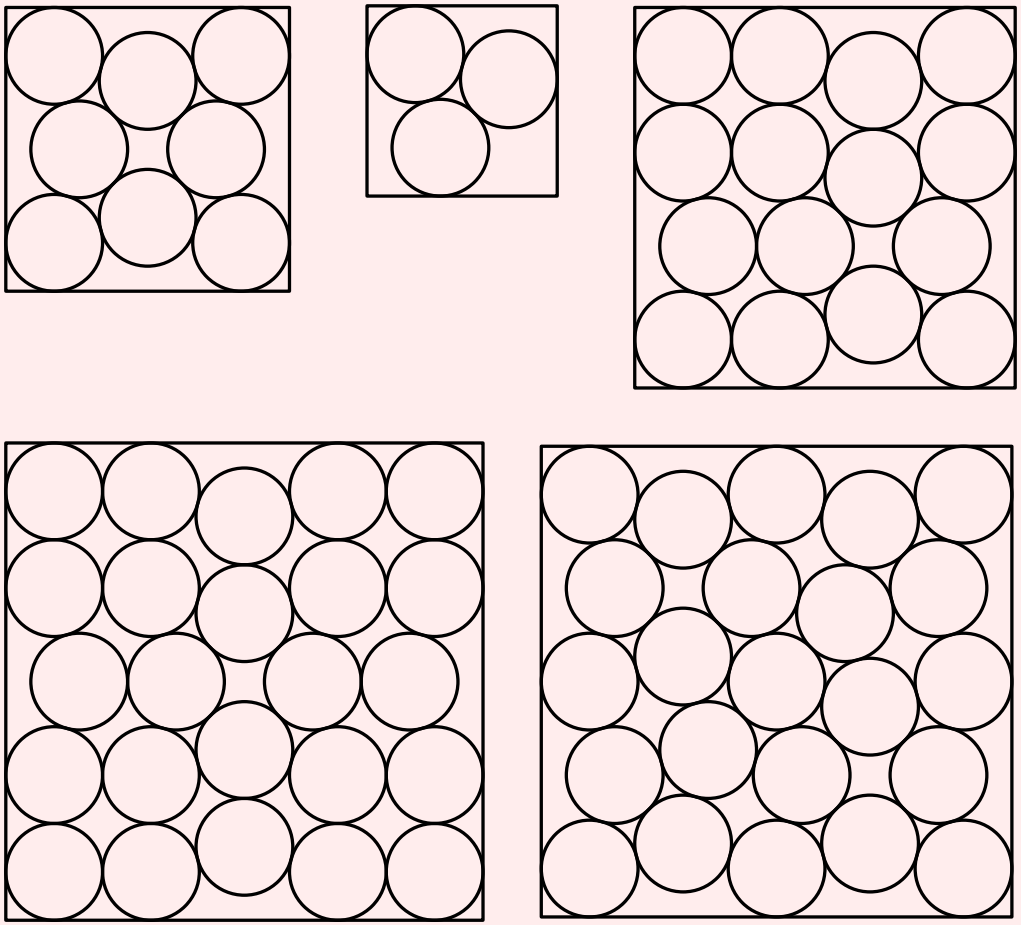
- Start organizing the results

Perfect squares



$$F_{\square} = F_T$$

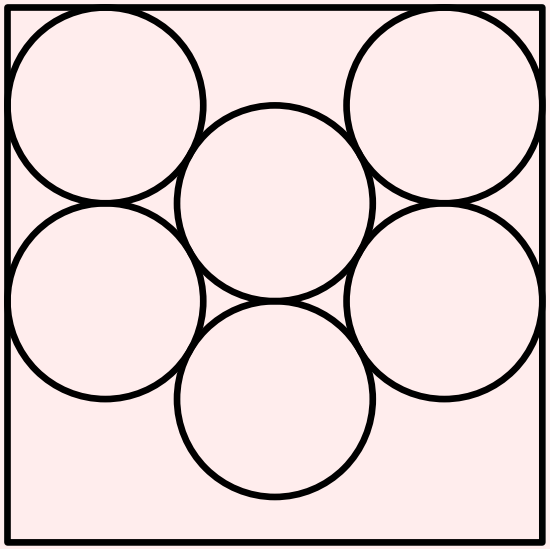
Perfect squares –1 or –2

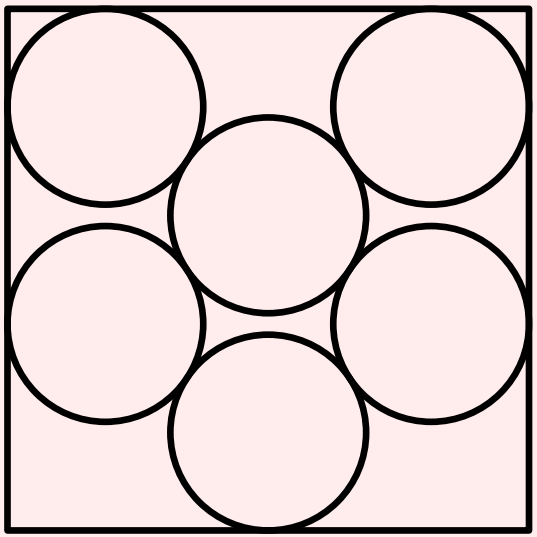


$F_8 \approx 0{,}731 \quad F_3 \approx 0{,}610 \quad F_{15} \approx 0{,}762$

$F_{24} \approx 0{,}775 \quad F_{23} \approx 0{,}764$

Squeezed horizontaly





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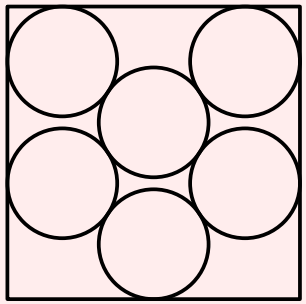
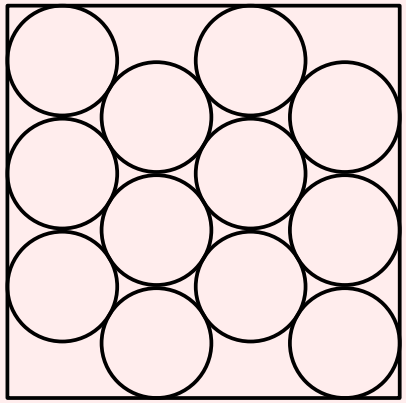
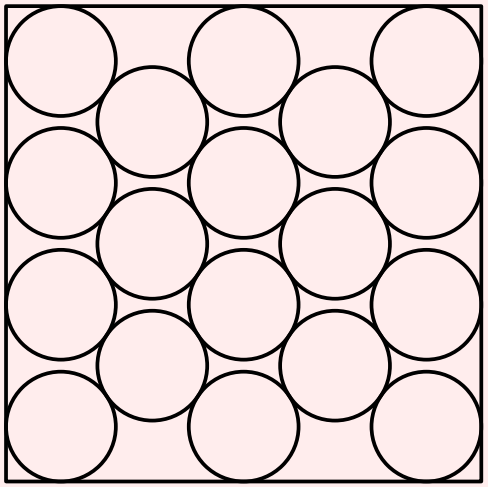
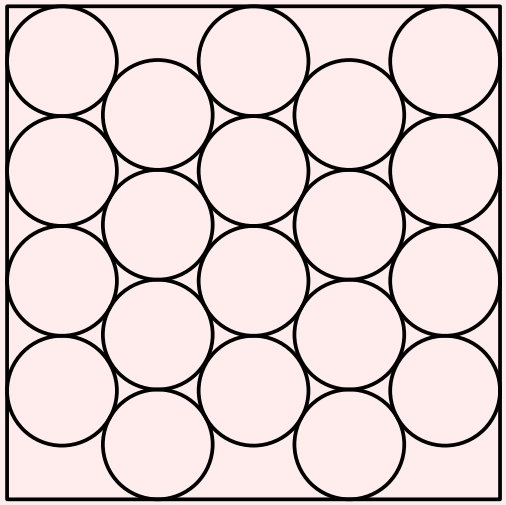




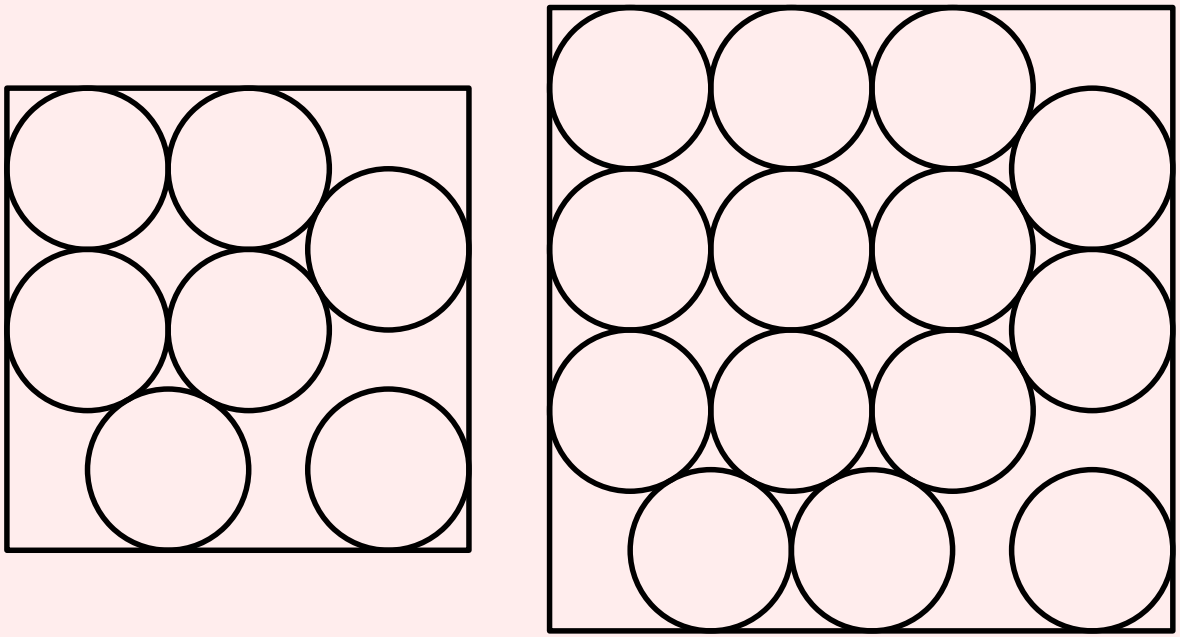




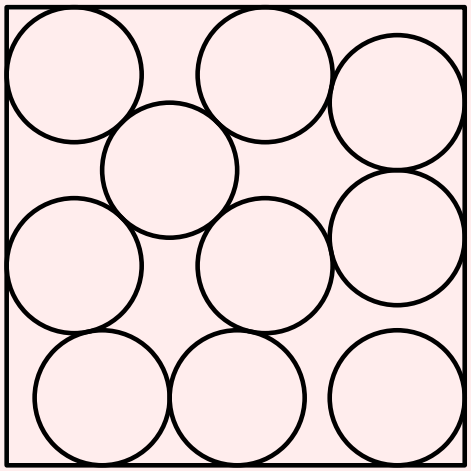
$$F_6 \approx 0{,}664 \quad F_{12} \approx 0{,}738$$

$$F_{18} \approx 0{,}755 \quad F_{20} \approx 0{,}780$$

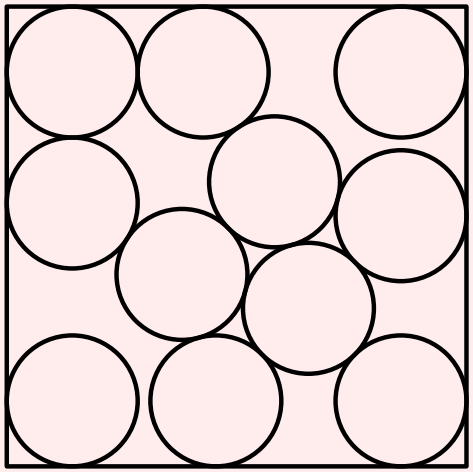
One free circle



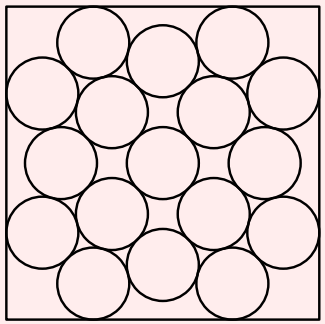
$F_7 \approx 0{,}670$ $F_{14} \approx 0{,}736$

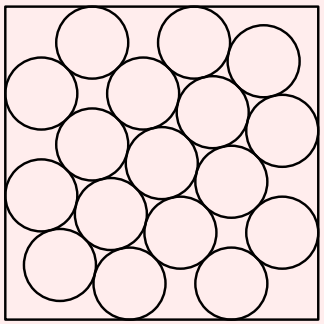


$F_{10} \approx 0{,}682$

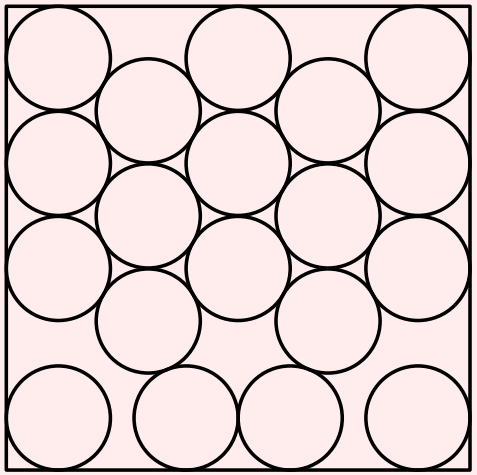


$F_{11} \approx 0{,}701$



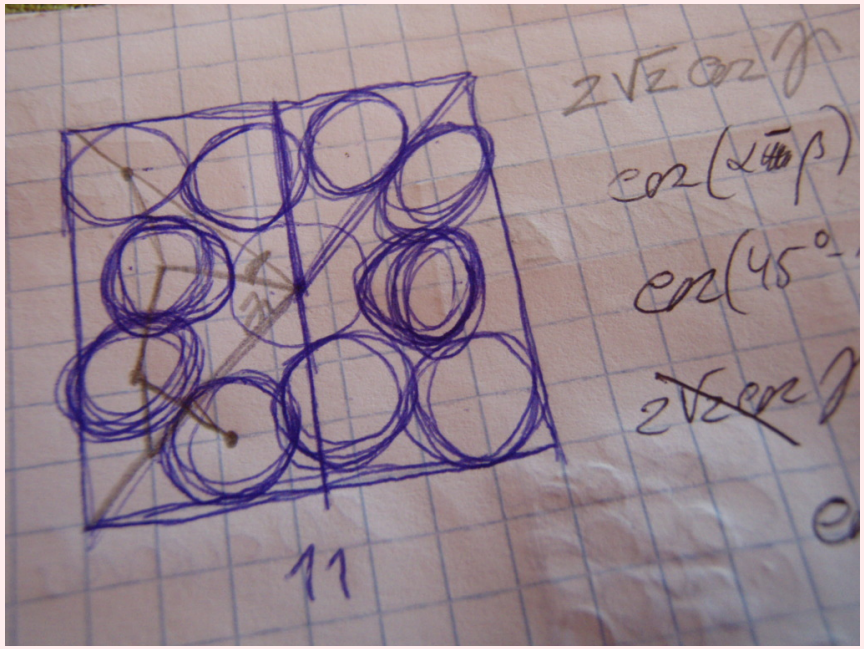


$F_{17} \approx 0{,}707$



$F_{19} \approx 0{,}752$

11



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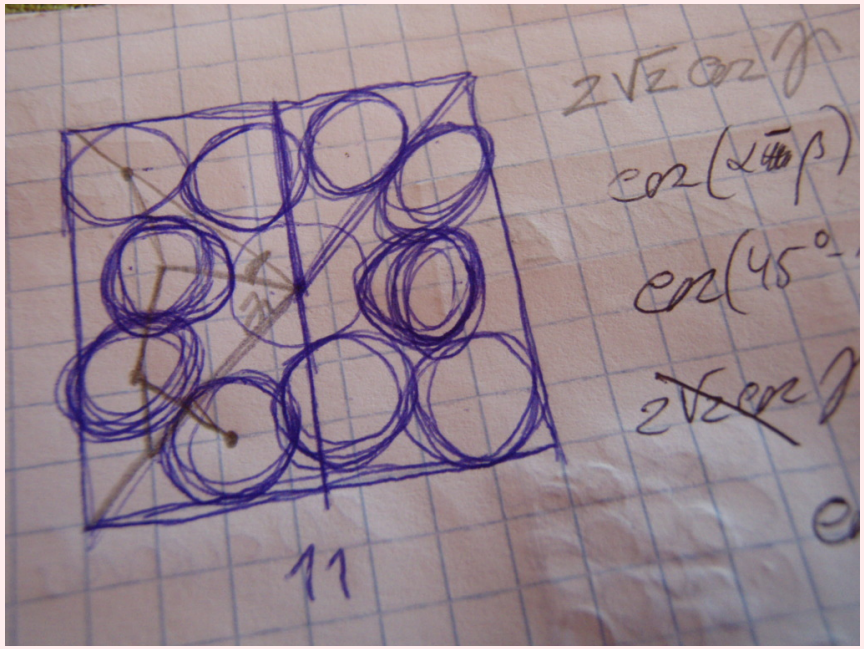
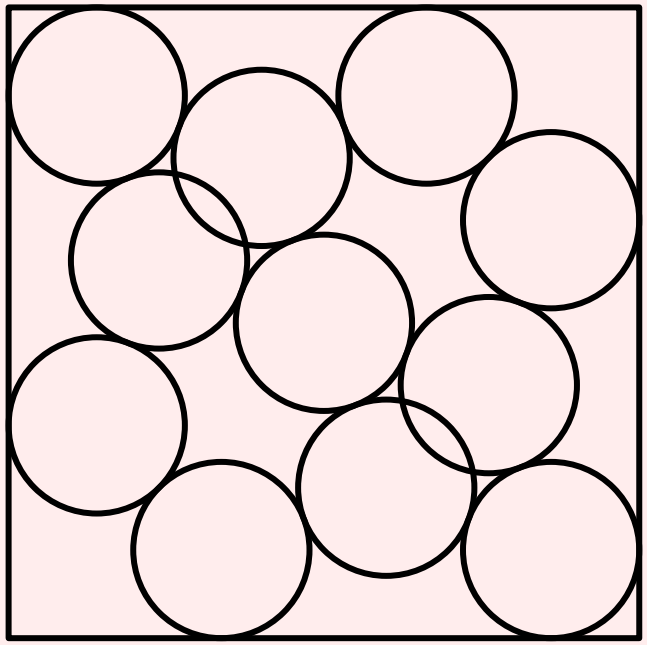
11

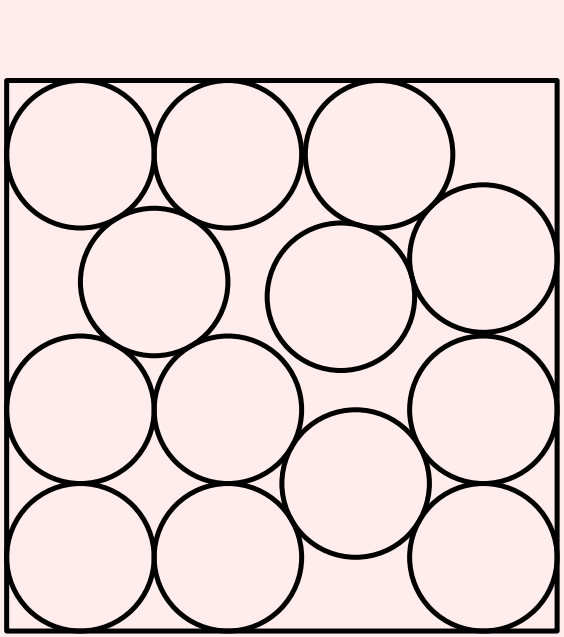


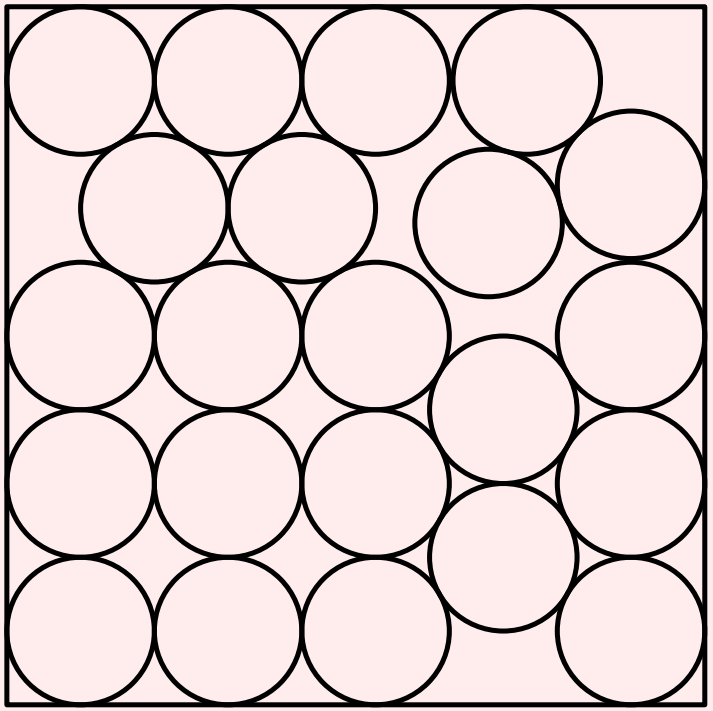


(SM)

- Make a simulation (it's cheaper than a real experiment)
- Constraints matter
- Different causes may have the same effect

Five free circles





$F_{13} \approx 0{,}733$ $F_{22} \approx 0{,}772$

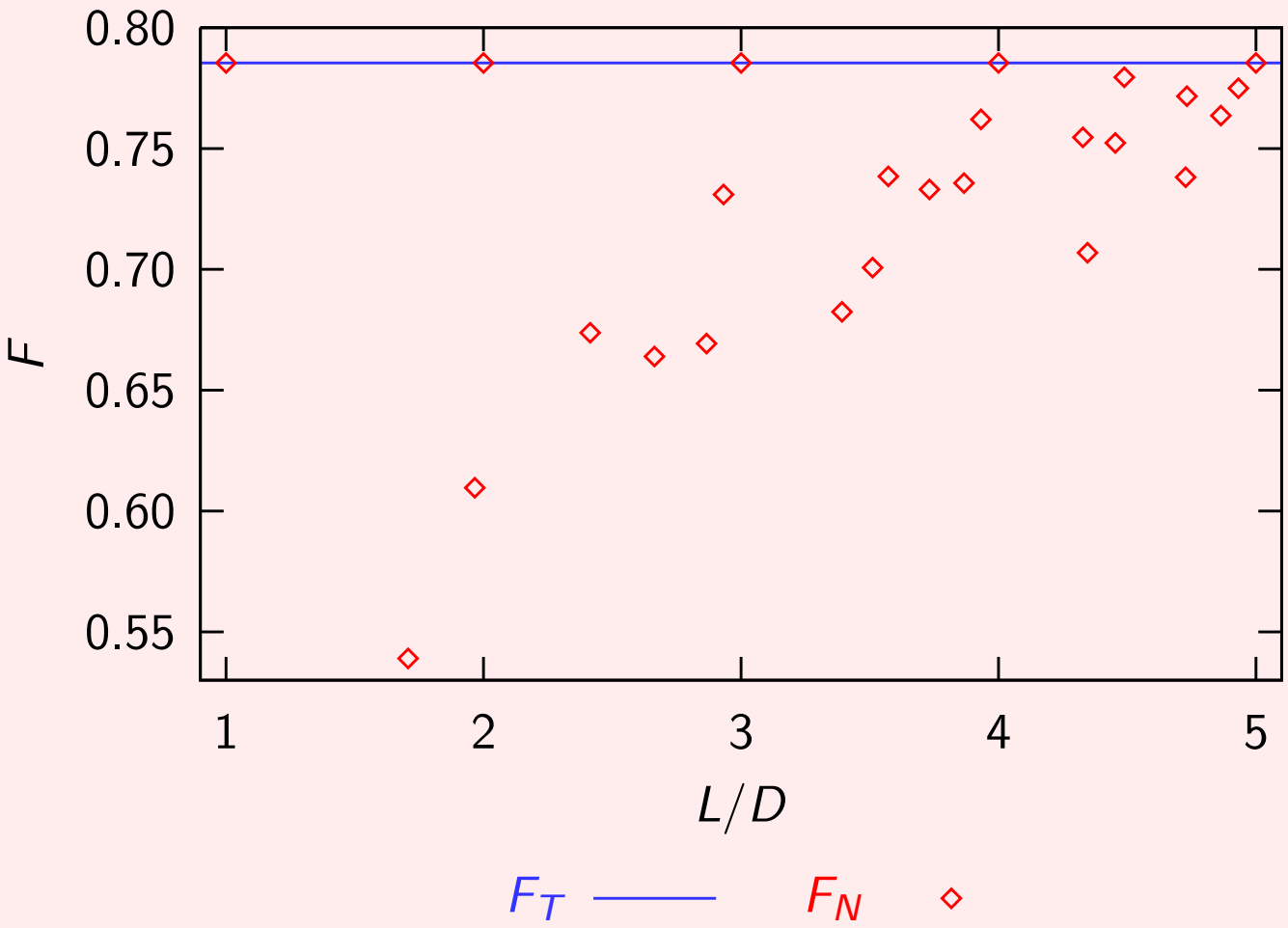
26

NO IDEA

(SM)

- Observe the big picture
- Pay attention to extremes

Big picture



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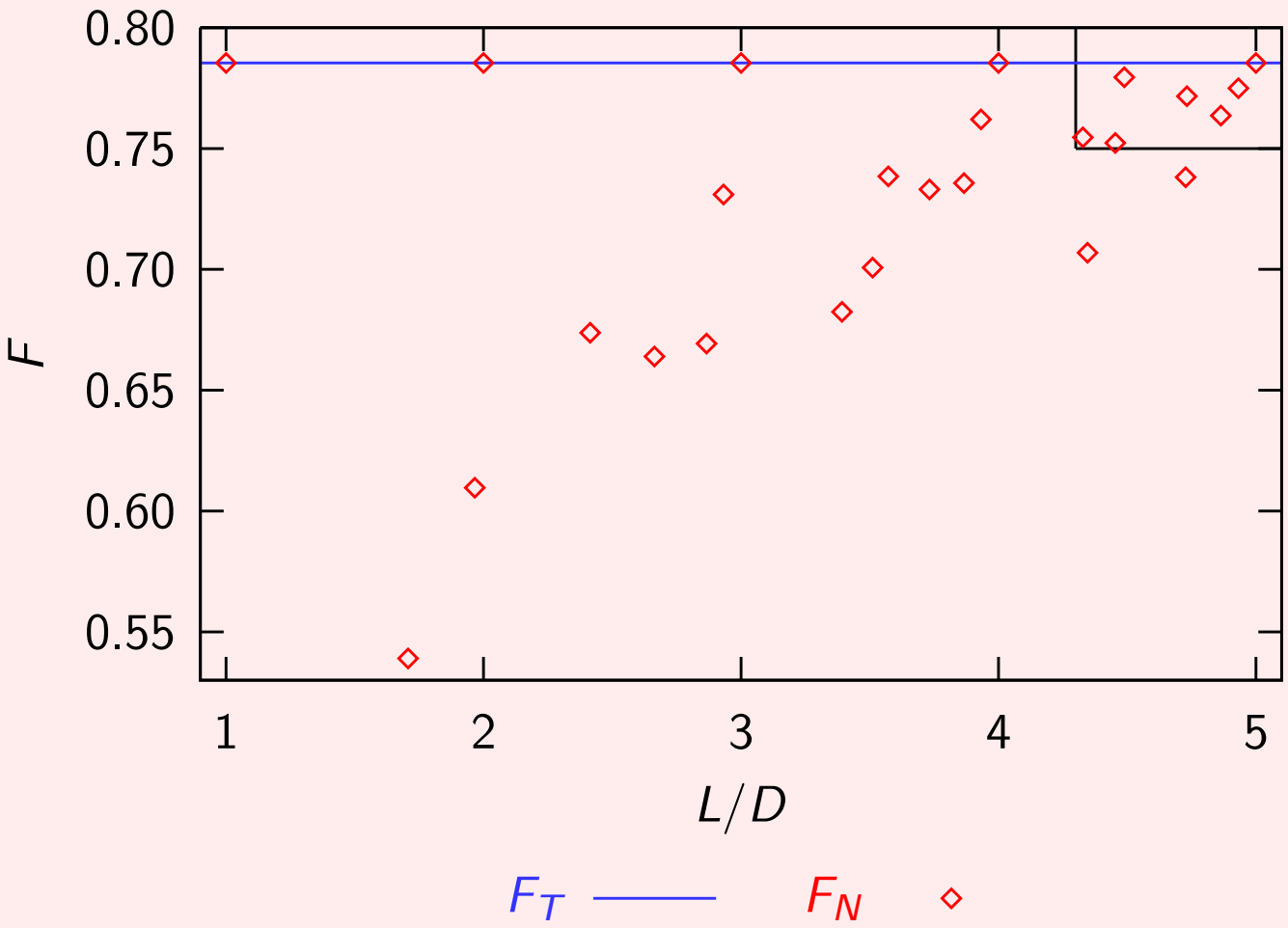
Big picture



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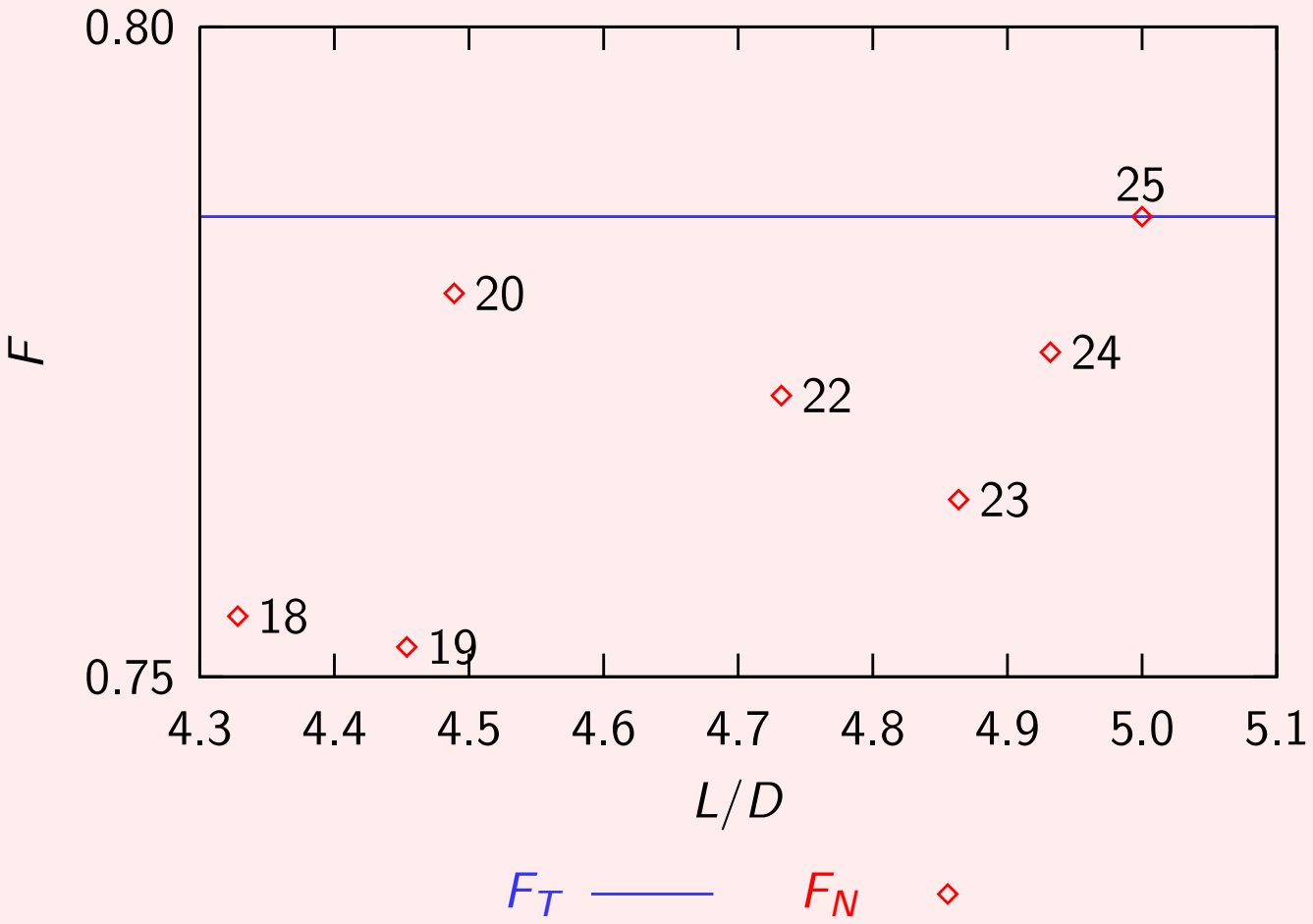
Extreme



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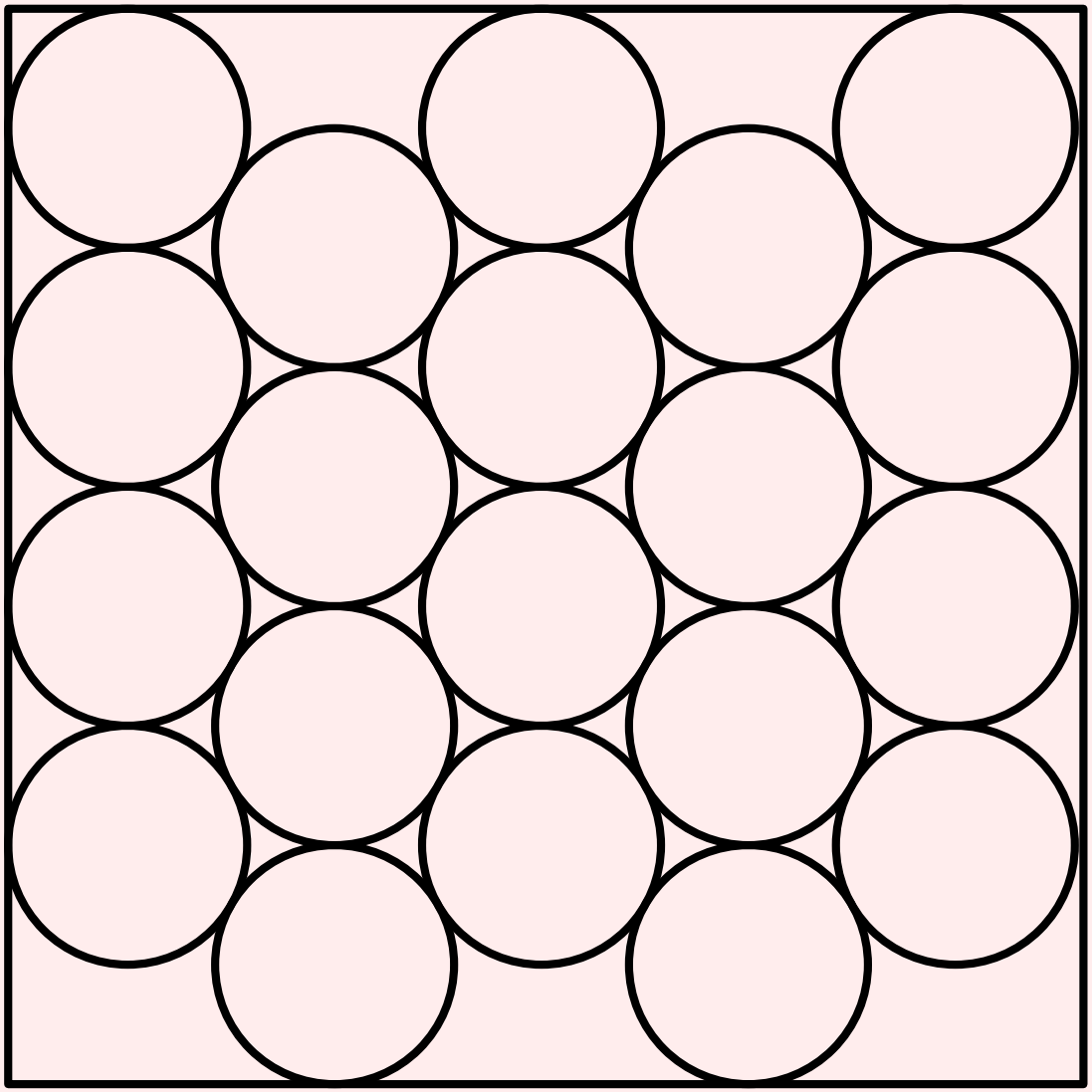
20



What is the smallest number of circles that can cover more area of a square than a single circle?
An example of the scientific method in action.

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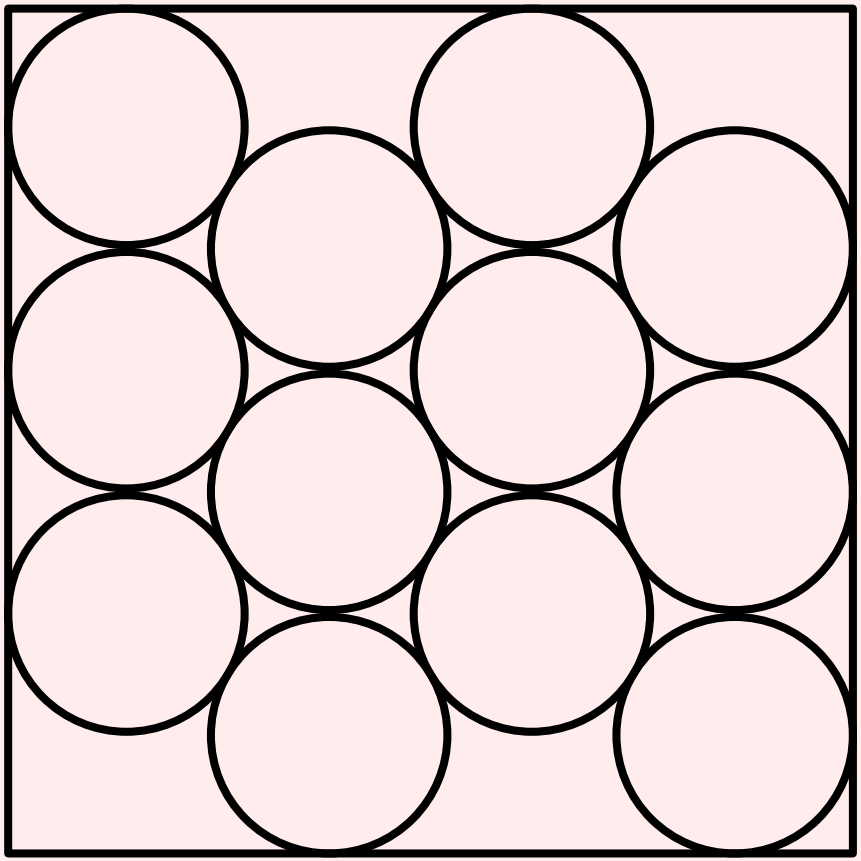
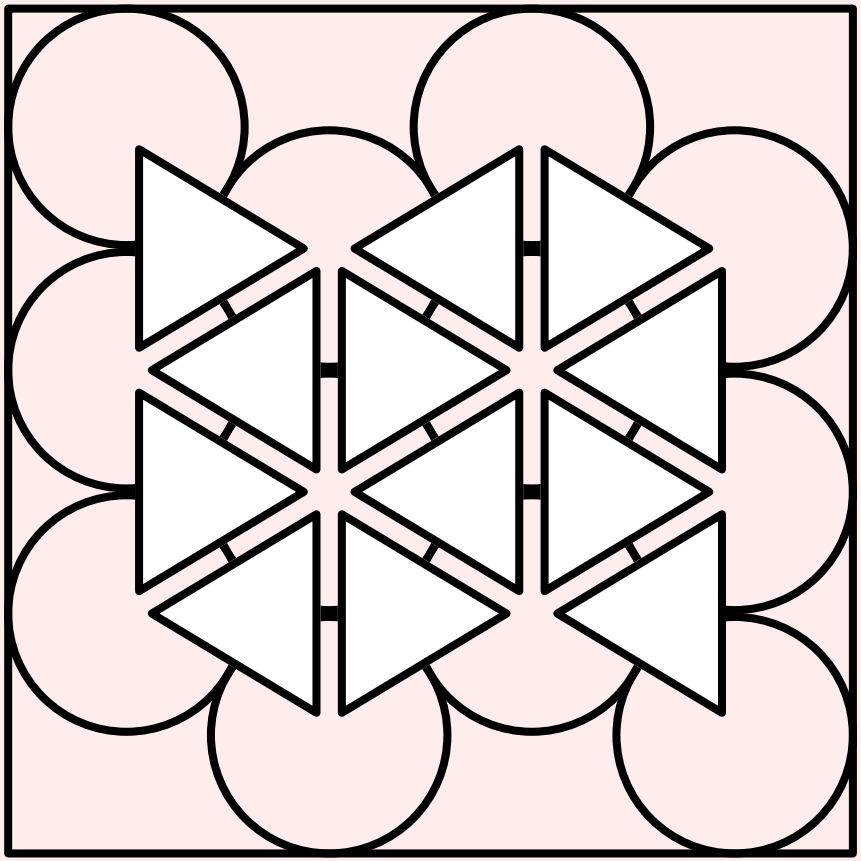
(SM)

- Do you see a pattern?

12



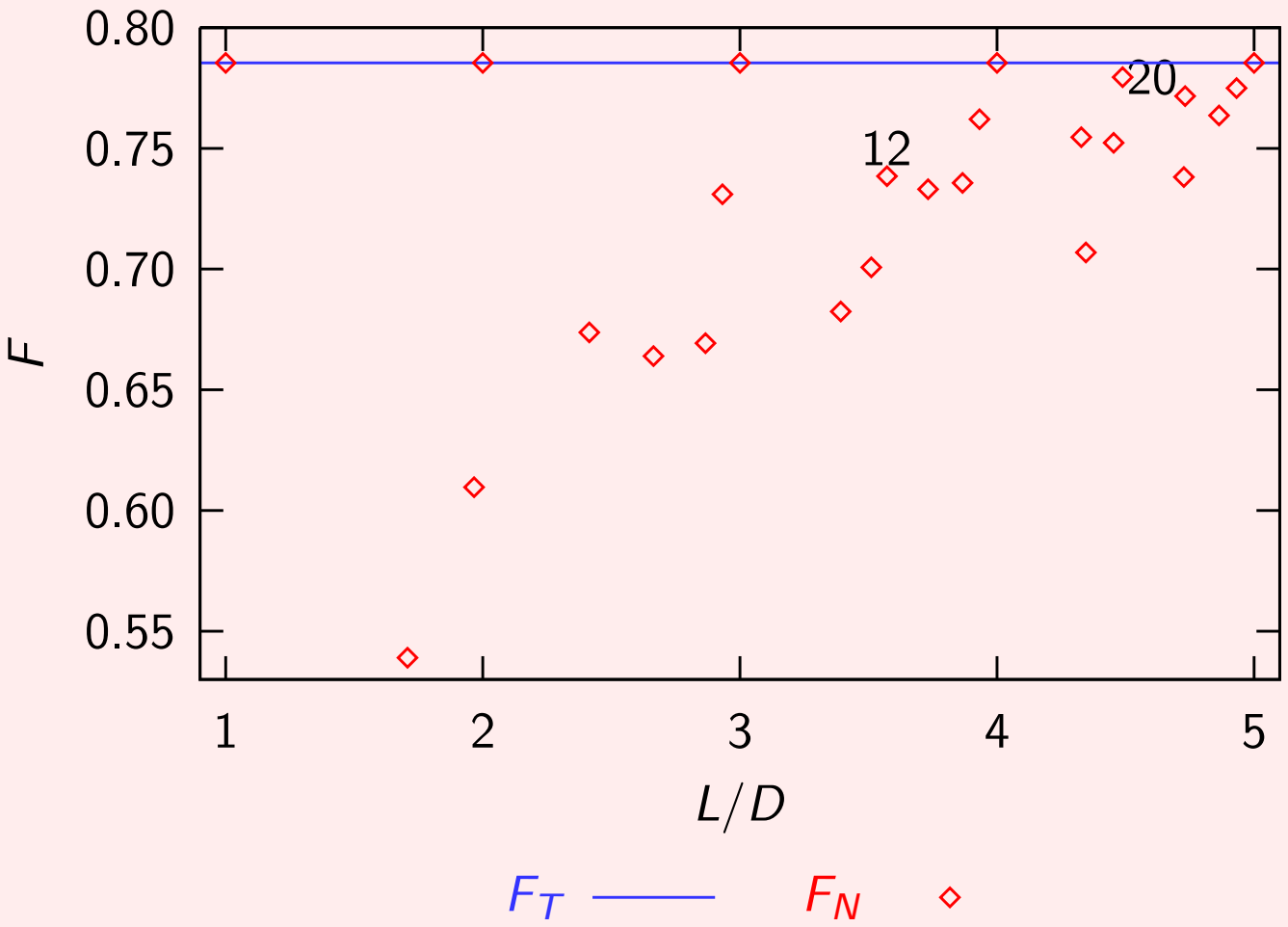
12



(SM)

- Confirm.

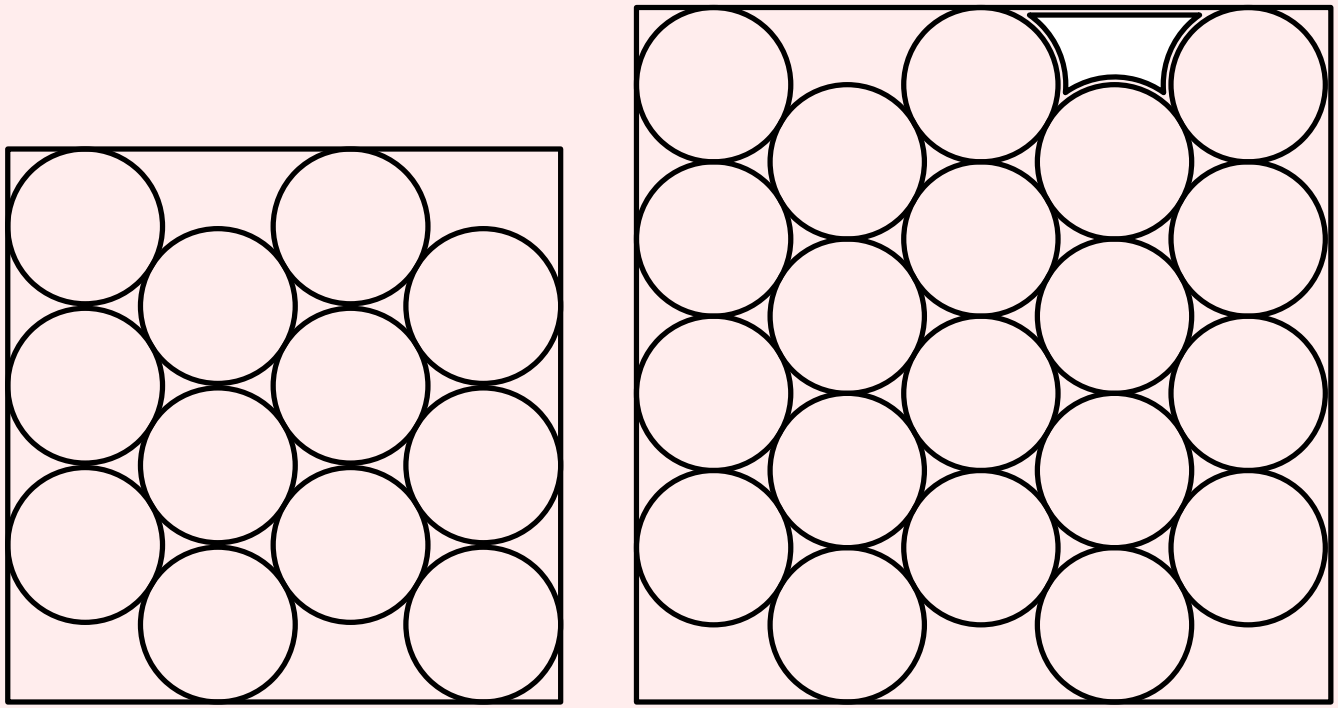
Both 20 and 12 are local maxima



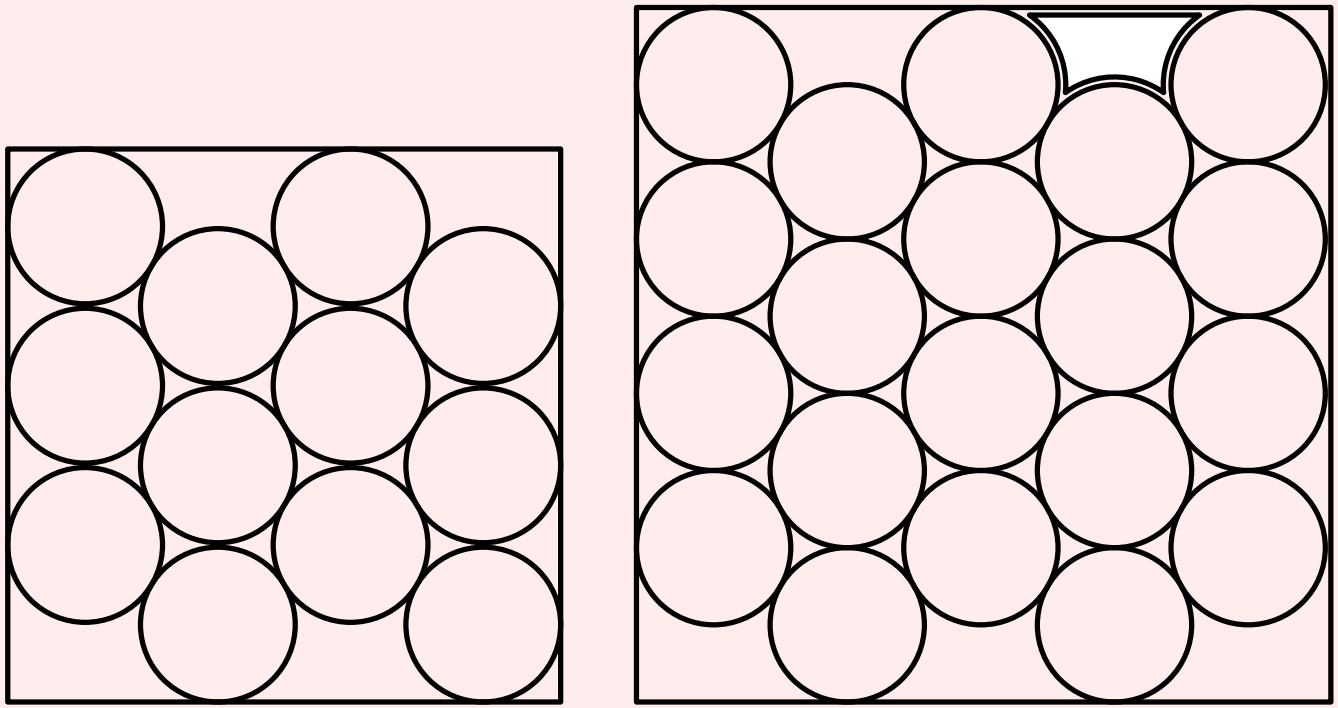
(SM)

- Why are our efforts insufficient upto now?
- Don't go on without knowing why.

Boundary effects



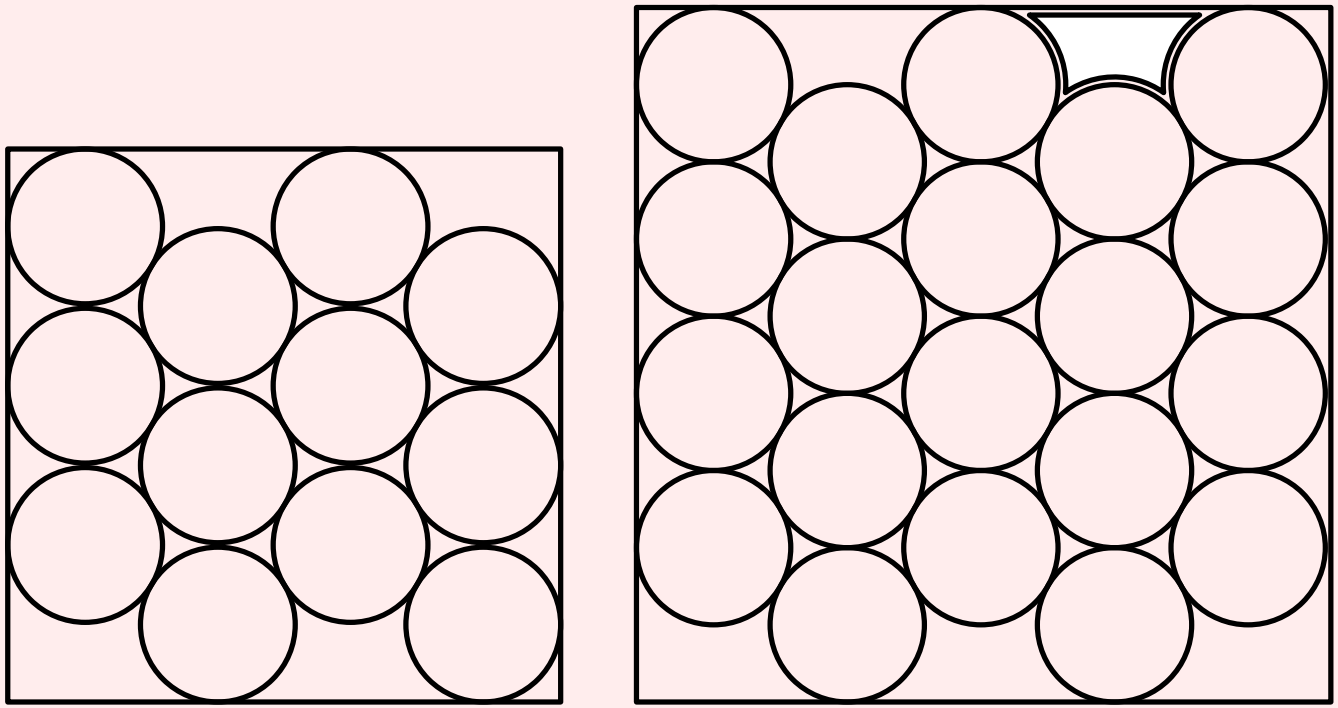
Boundary effects



Big voids at the boundary: $+1$

Circles: $+8$

Boundary effects



Big voids at the boundary: $+1$

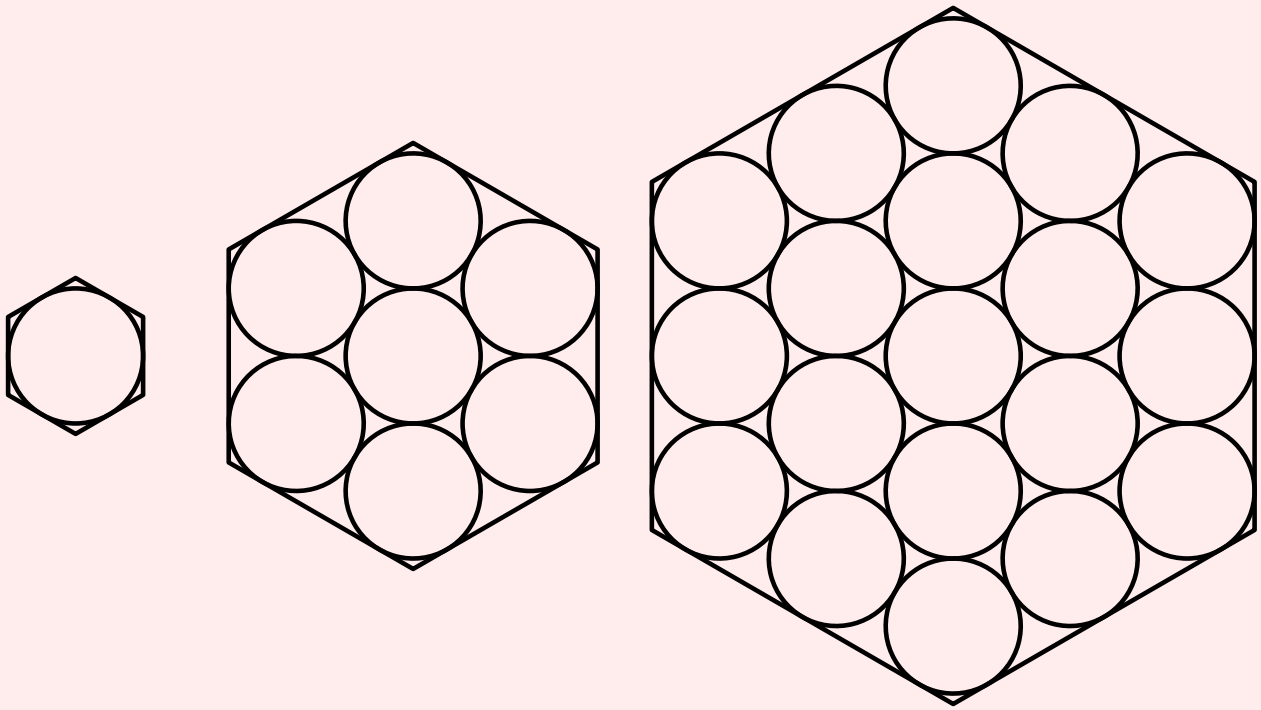
Circles: $+8$

$$N \nearrow \quad \Rightarrow \quad \text{voids/circle} \searrow$$

(SM)

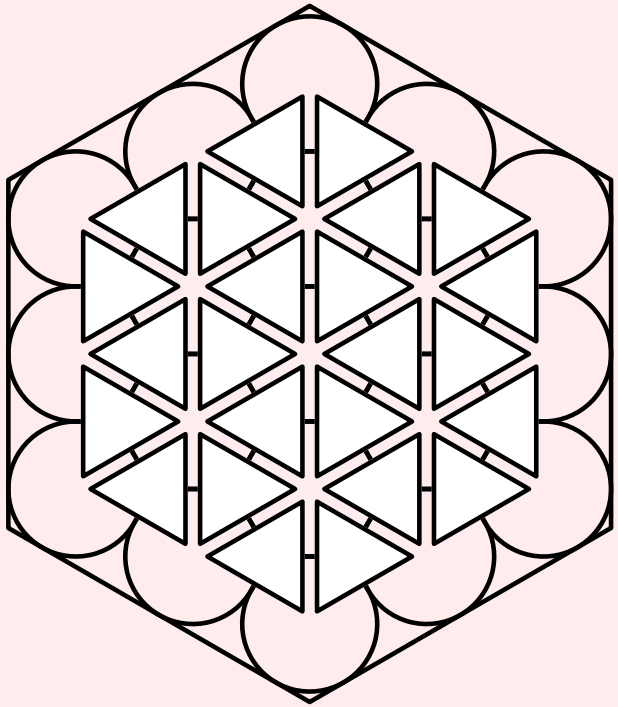
- Using only this pattern, can we expect to succeed?

Hexagonal configuration



$$F_{max} = \frac{\pi}{2\sqrt{3}} \approx 0{,}907$$

Hexagonal configuration

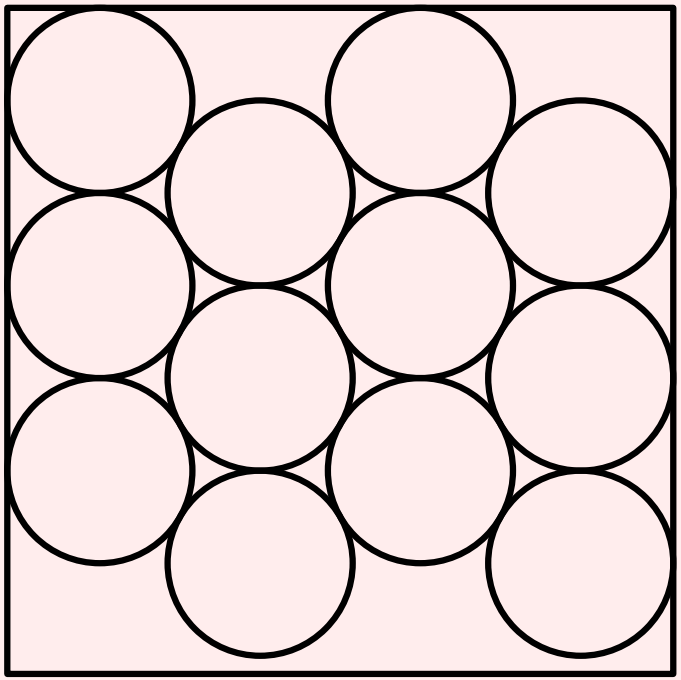


$$F_{max} = \frac{\pi}{2\sqrt{3}} \approx 0{,}907$$

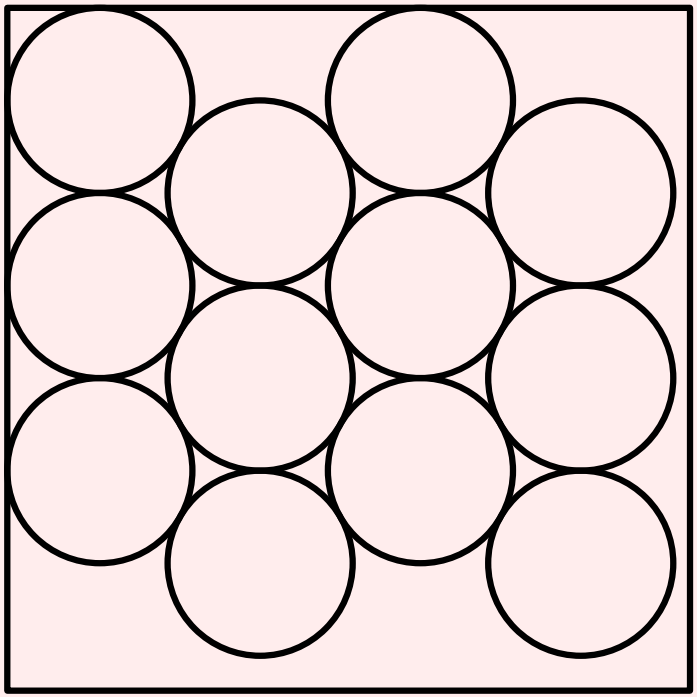
(SM)

- Could we have skipped the initial work?

Simulation



Simulation



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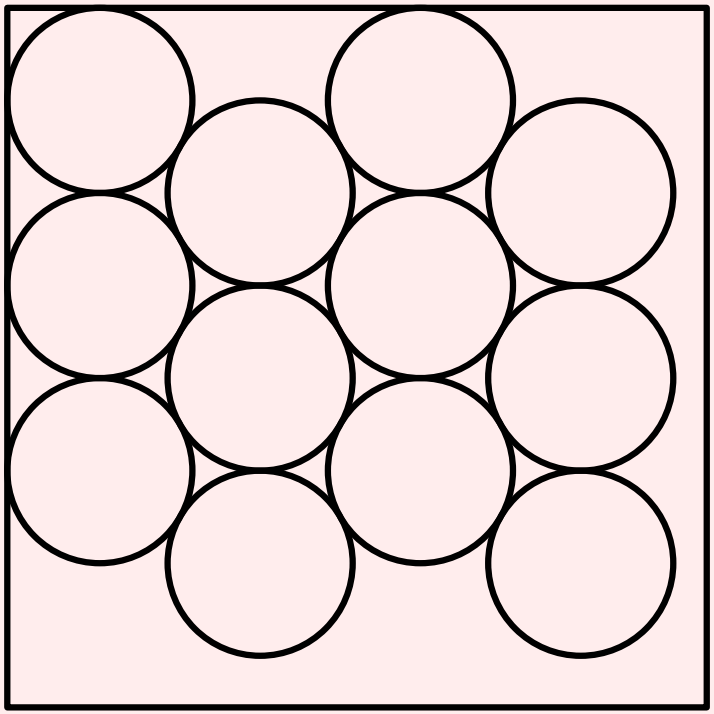

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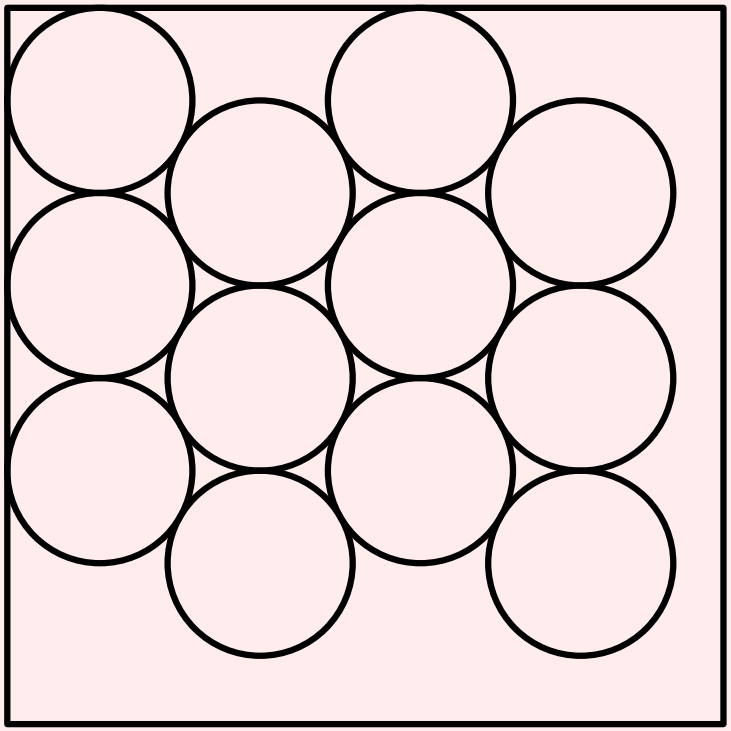
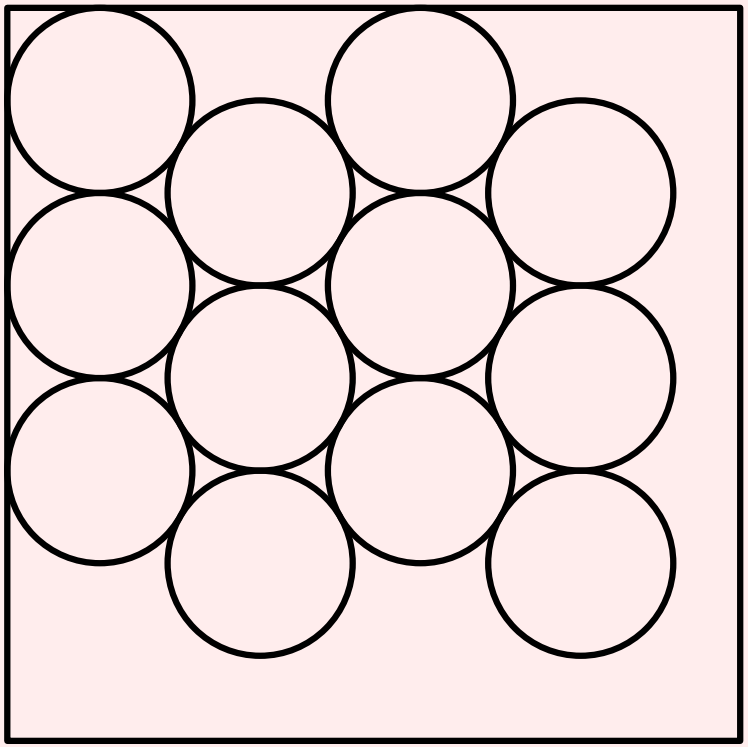


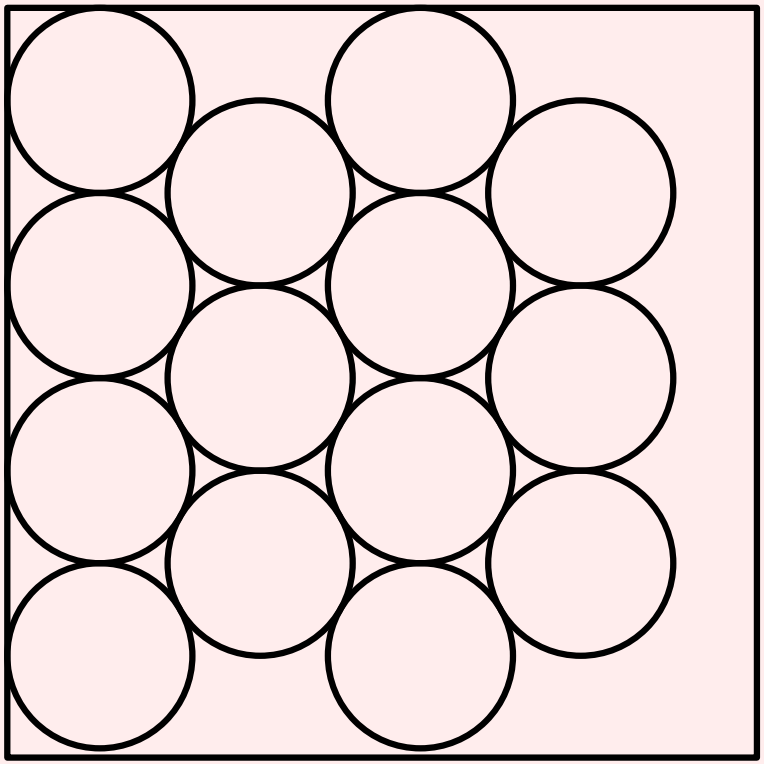
12

Simulation



12

Simulation



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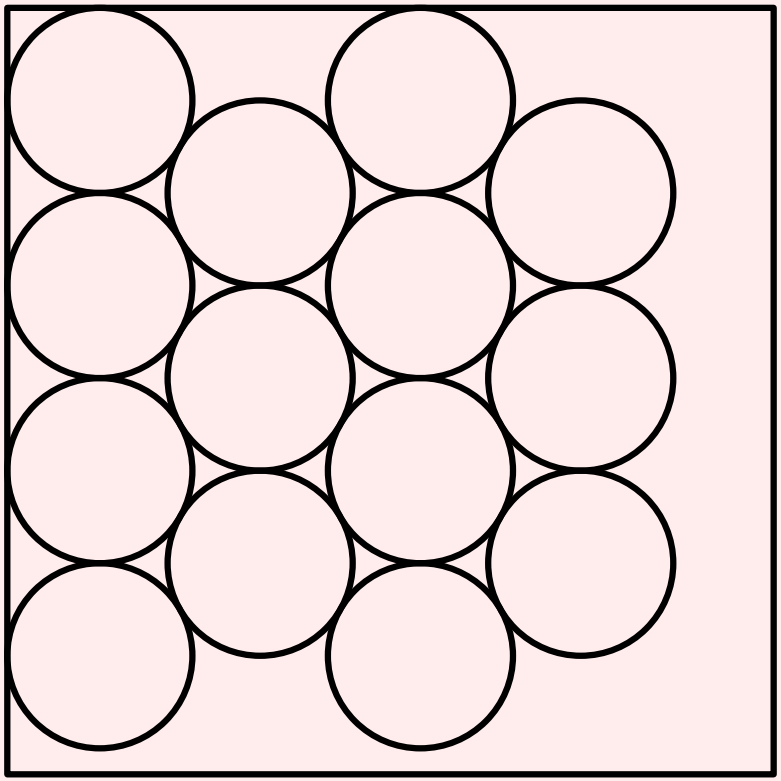
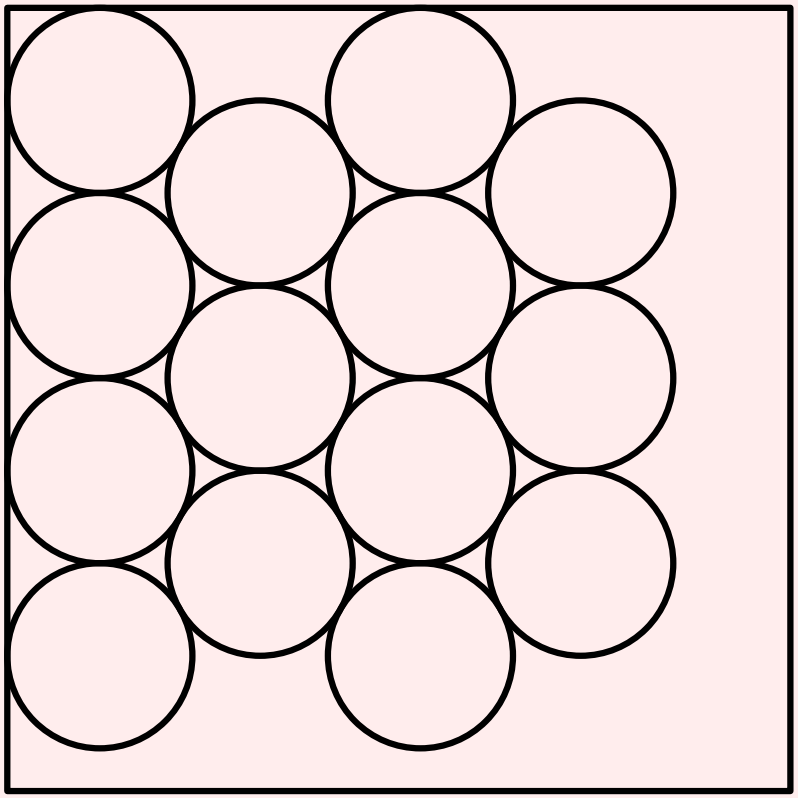


Simulation



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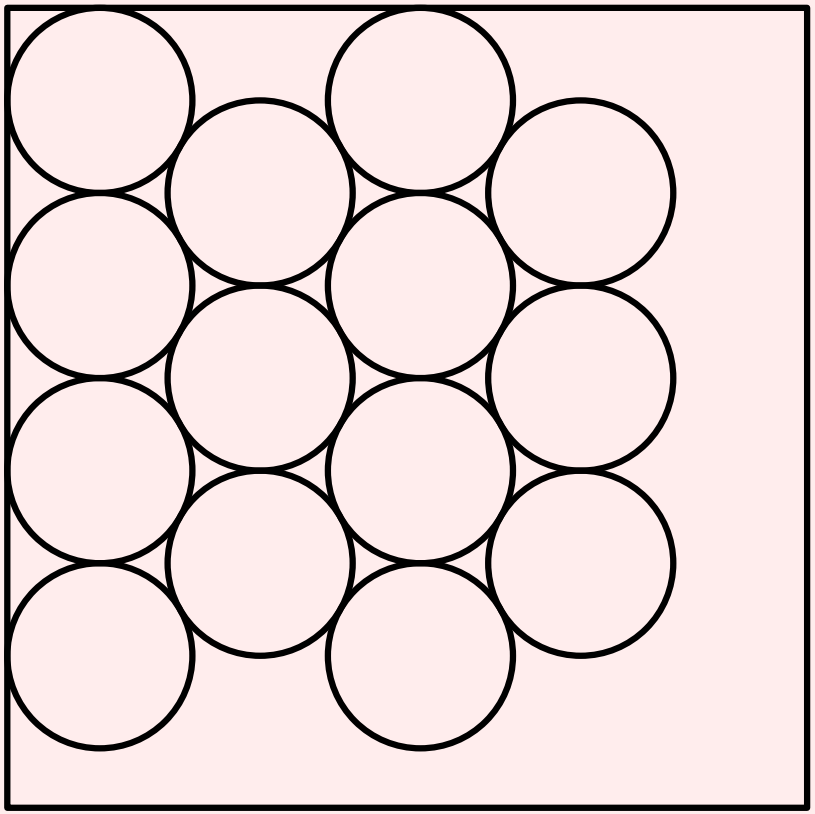

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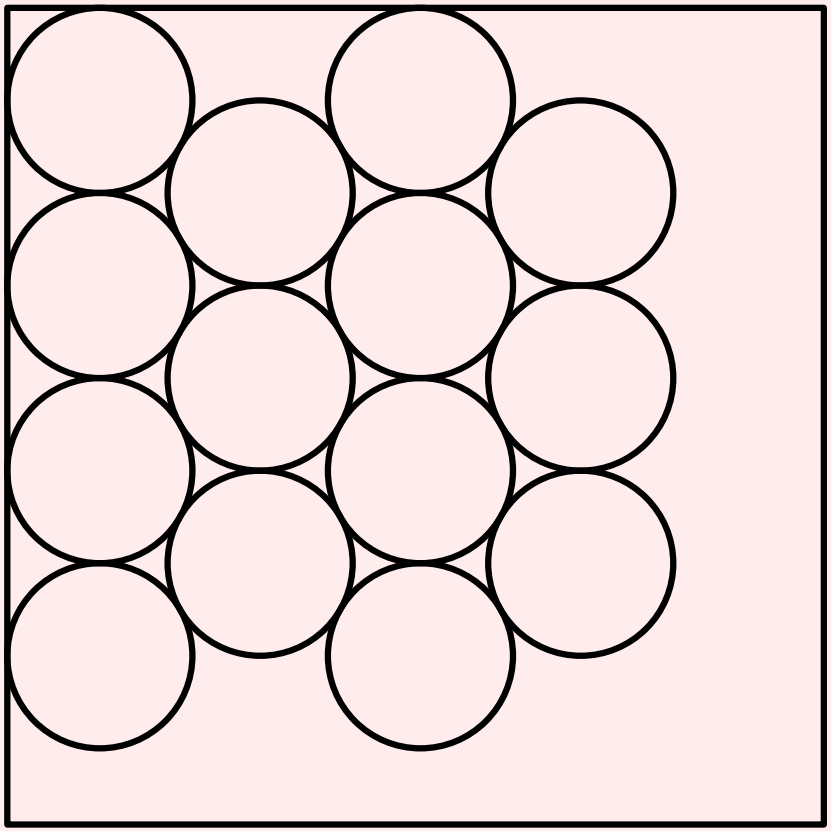

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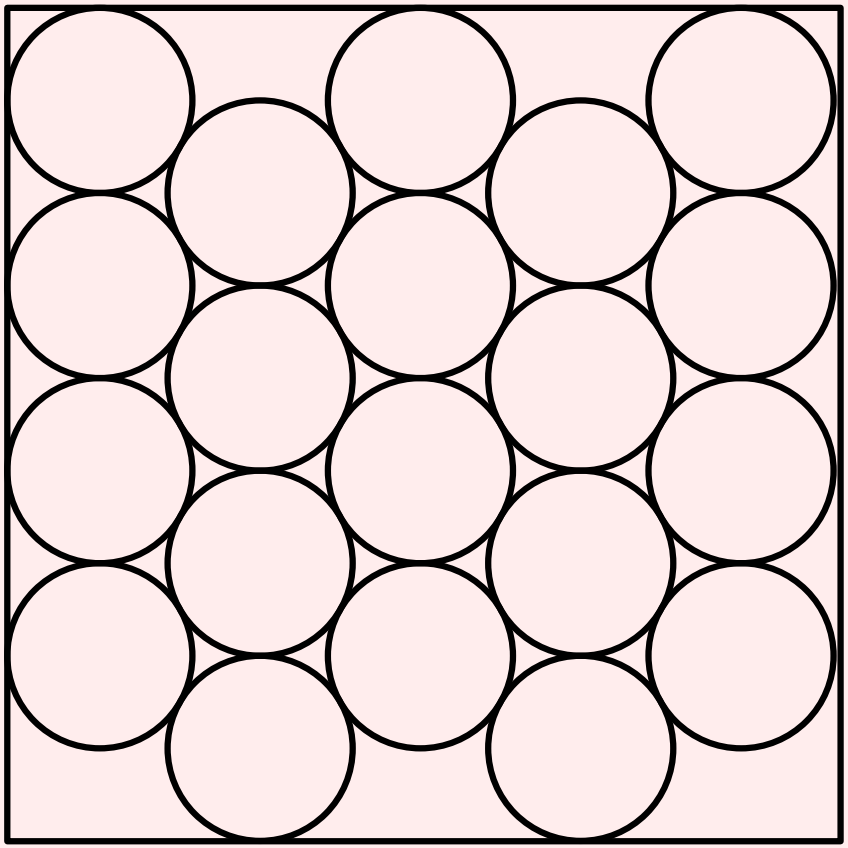

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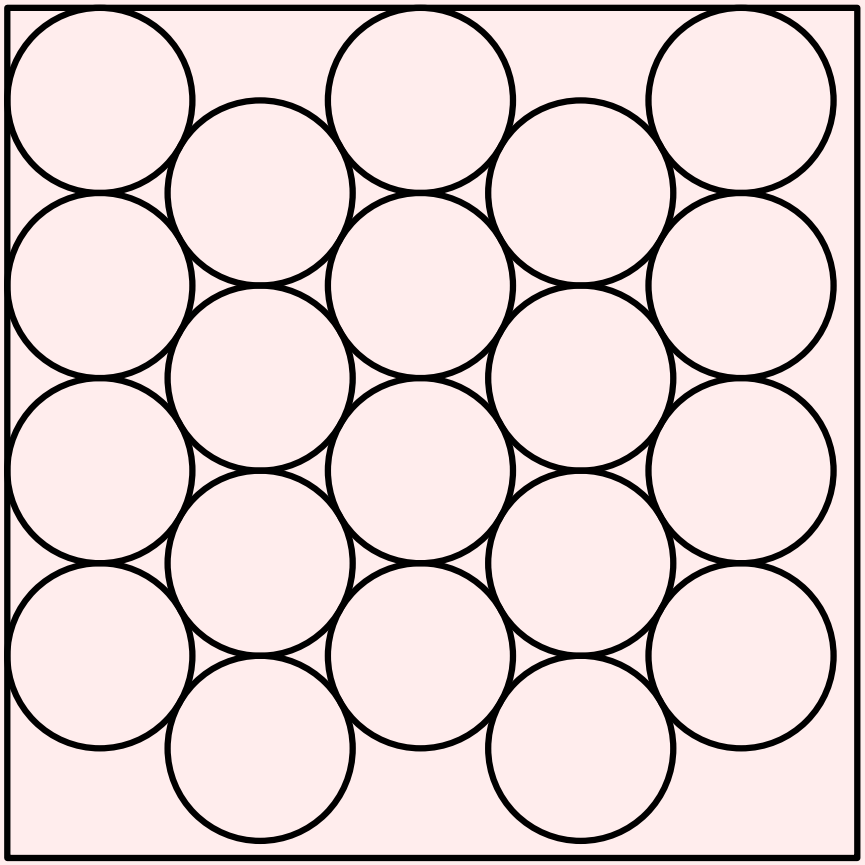

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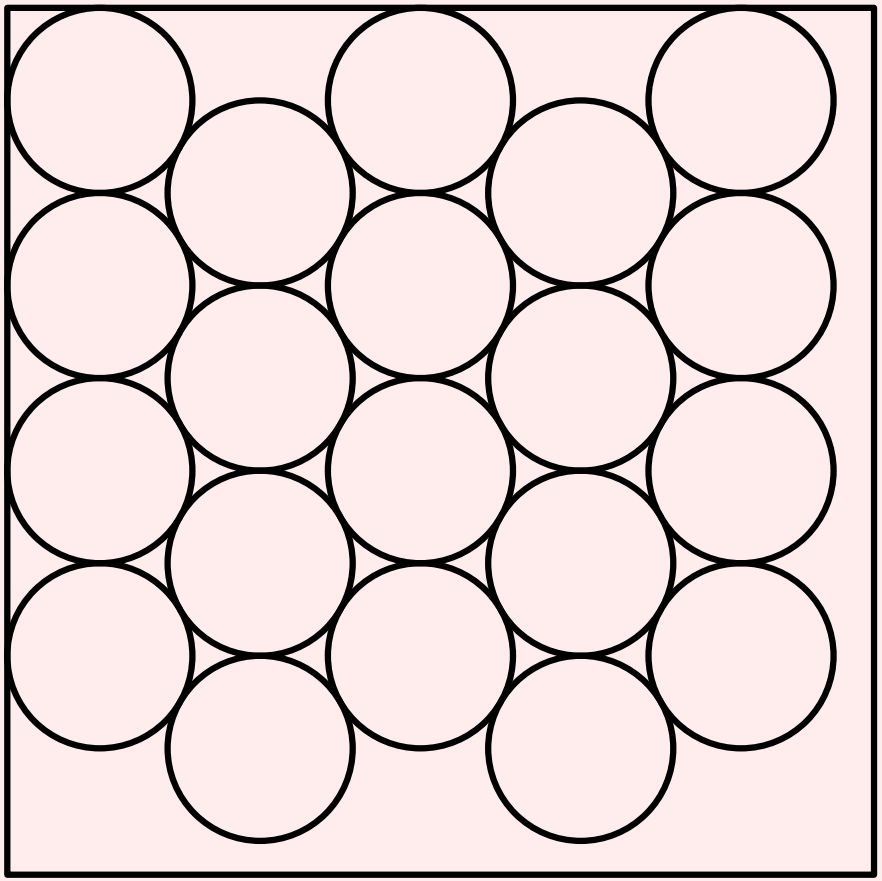
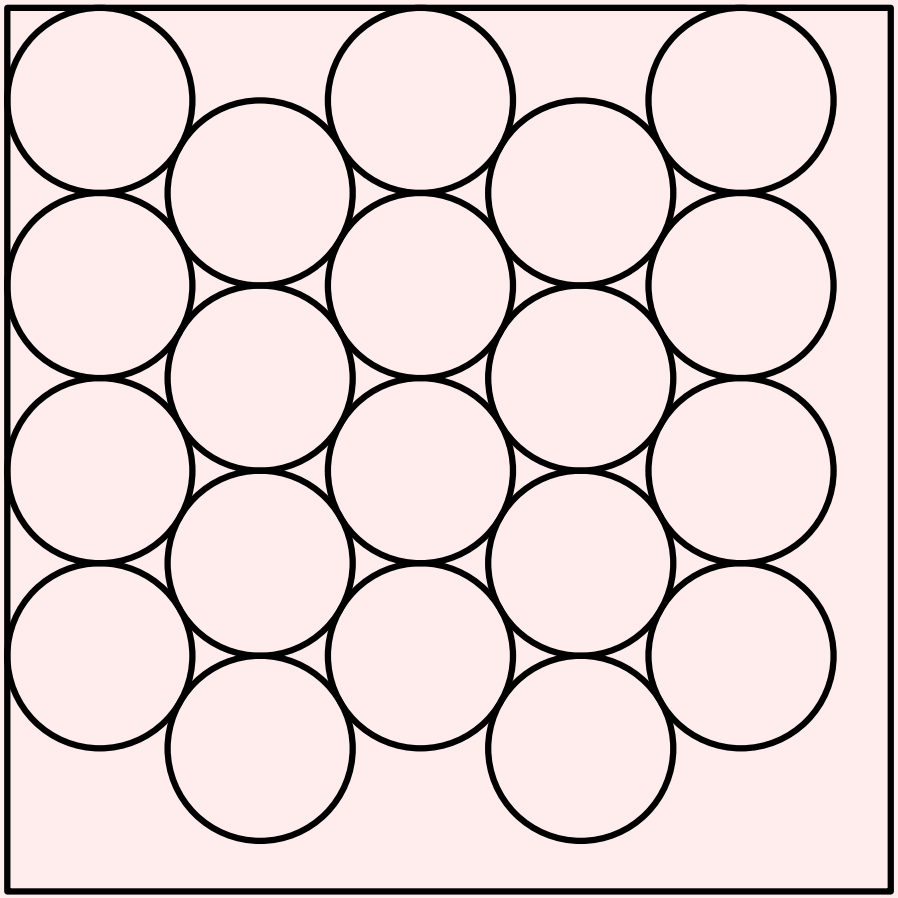
Simulation



Simulation



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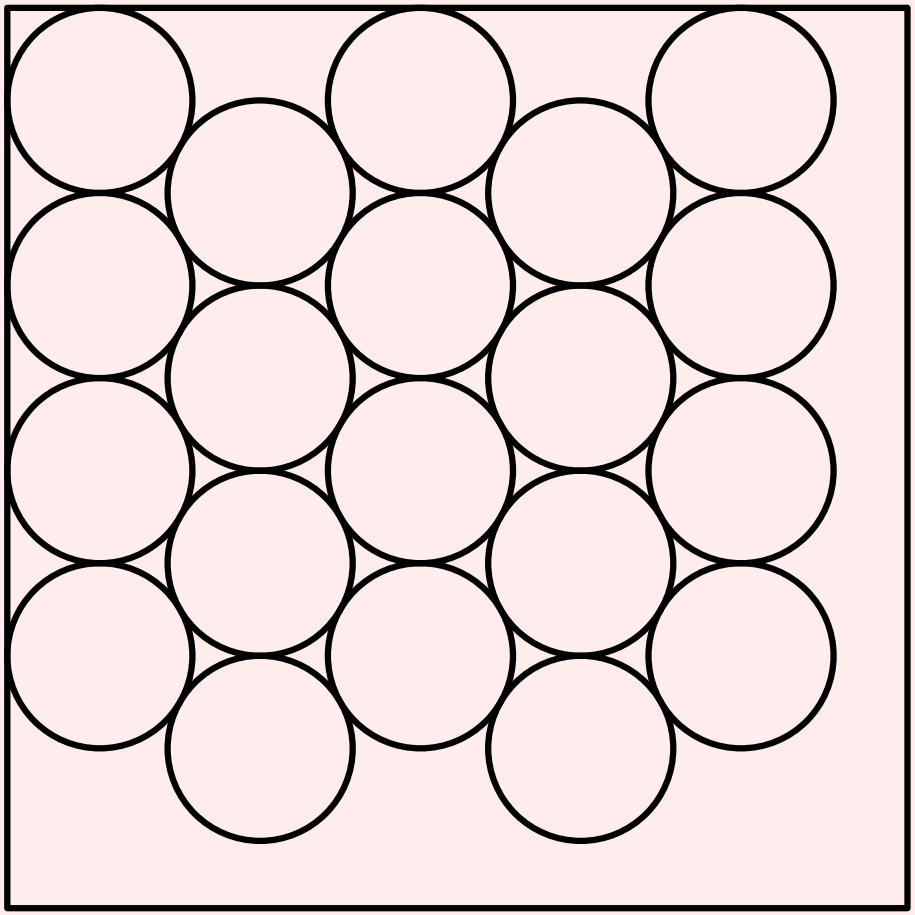

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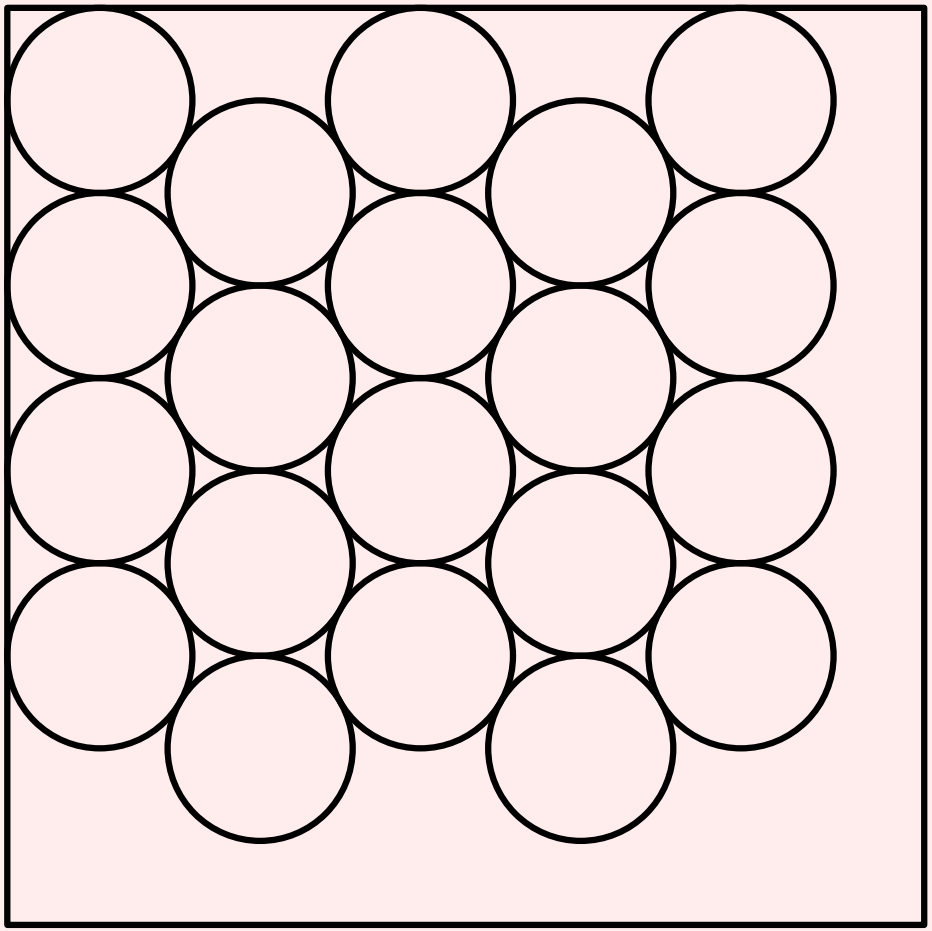

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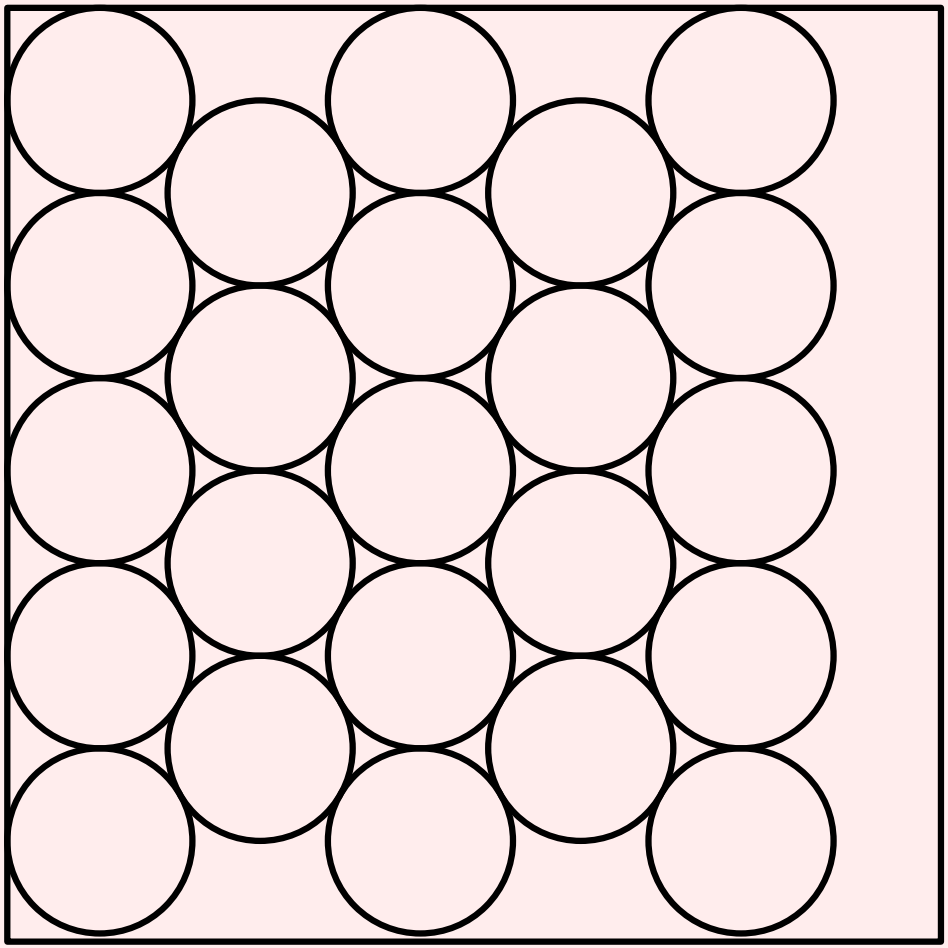
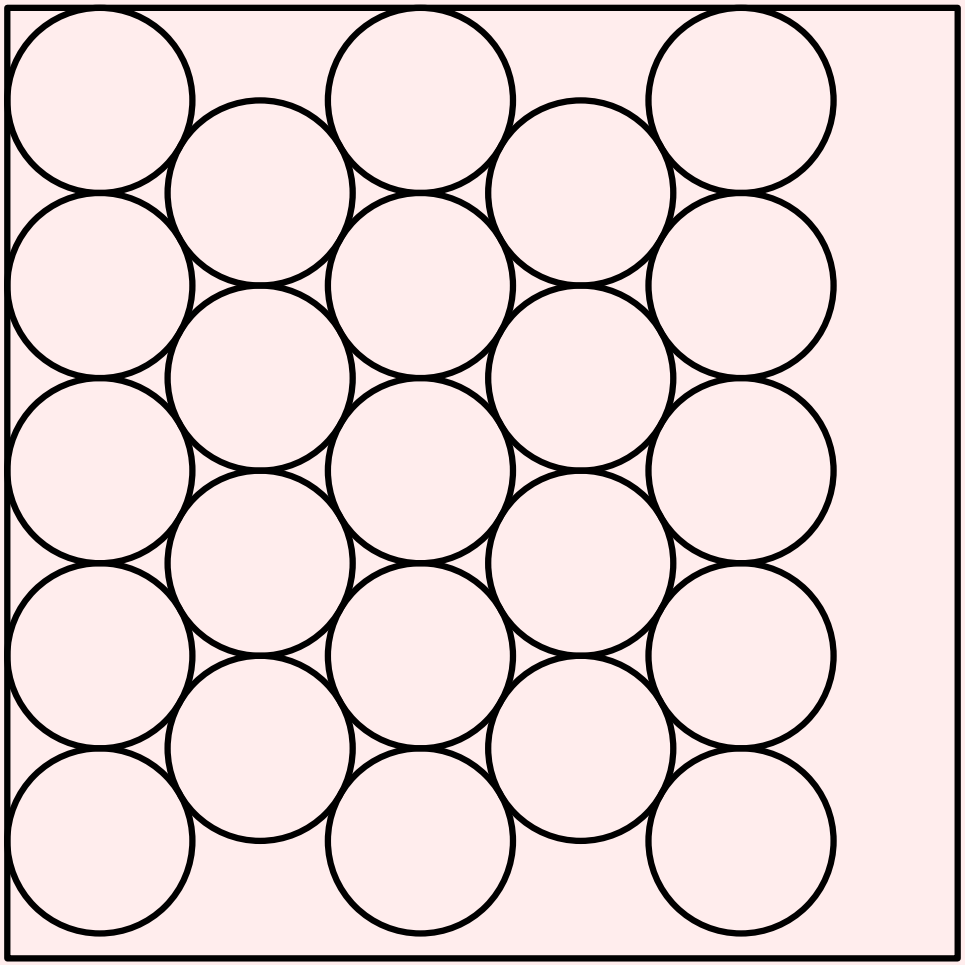
Simulation



Simulation



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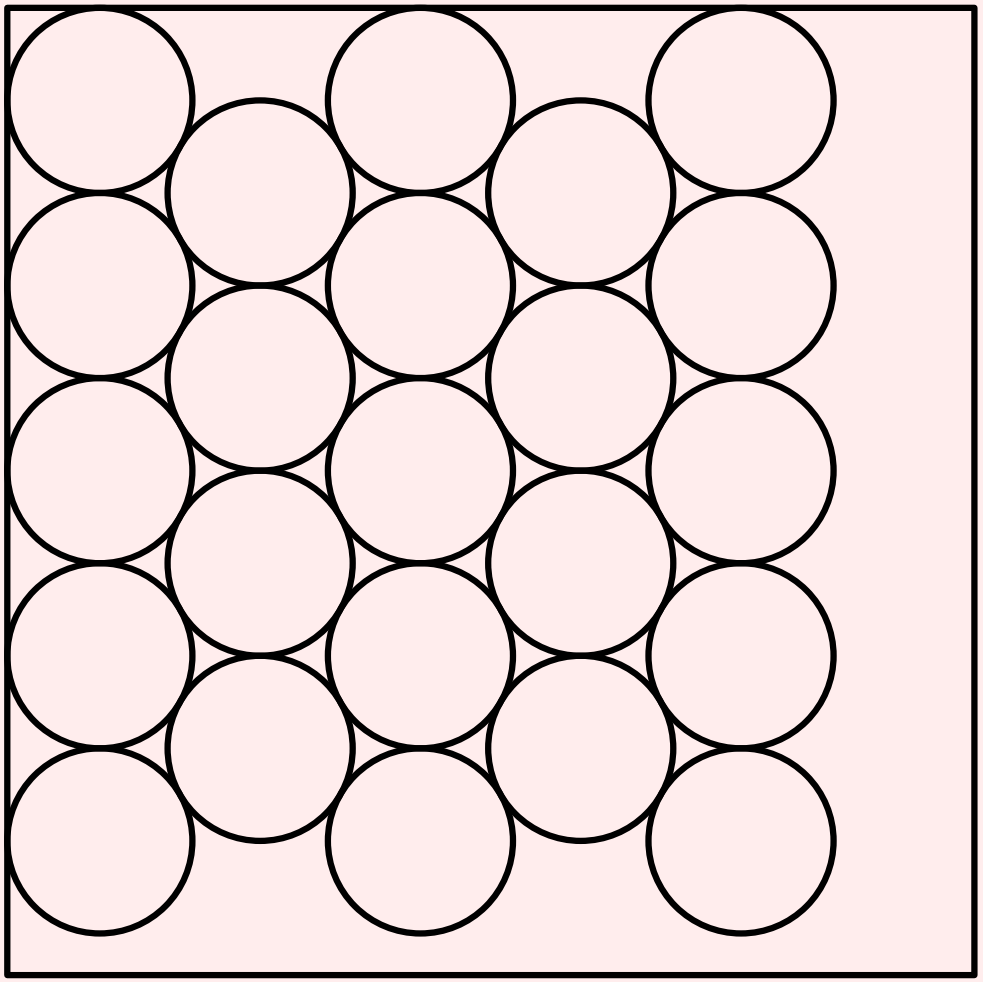

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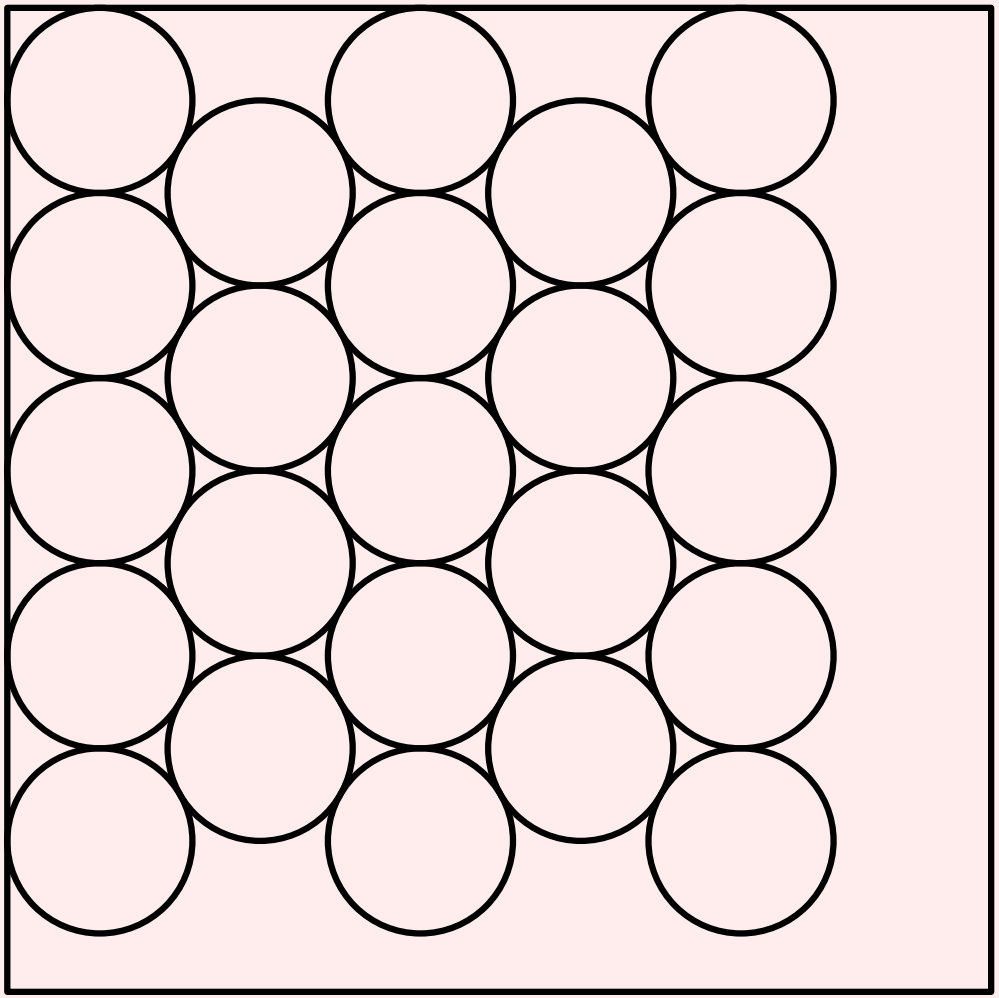

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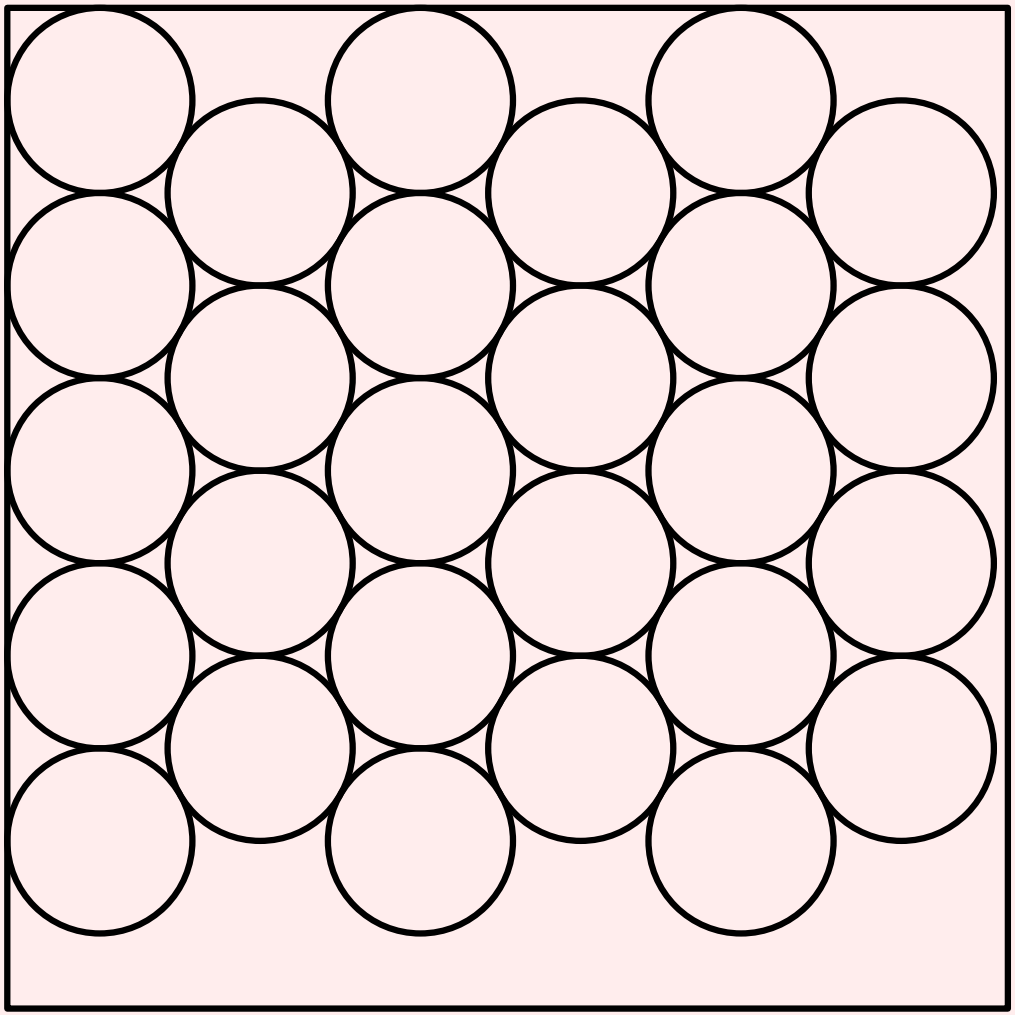

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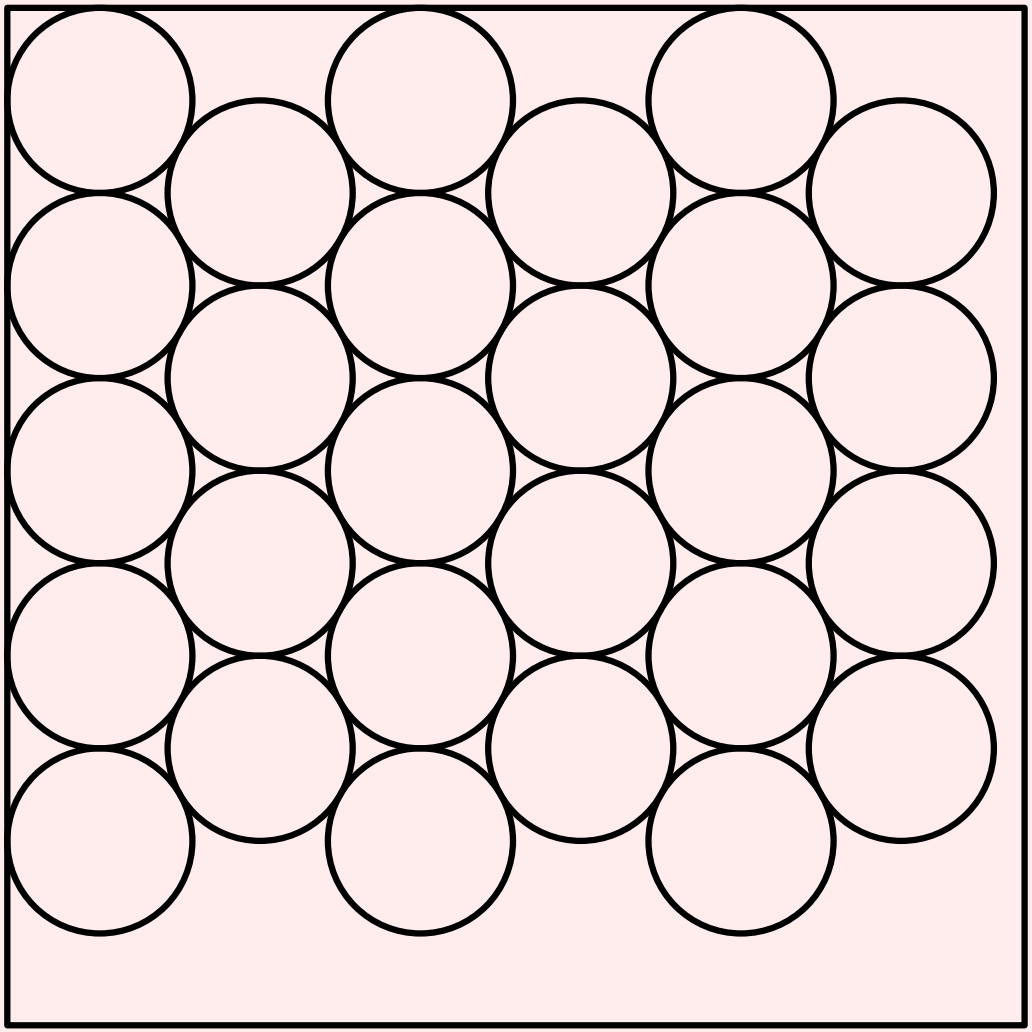
Simulation



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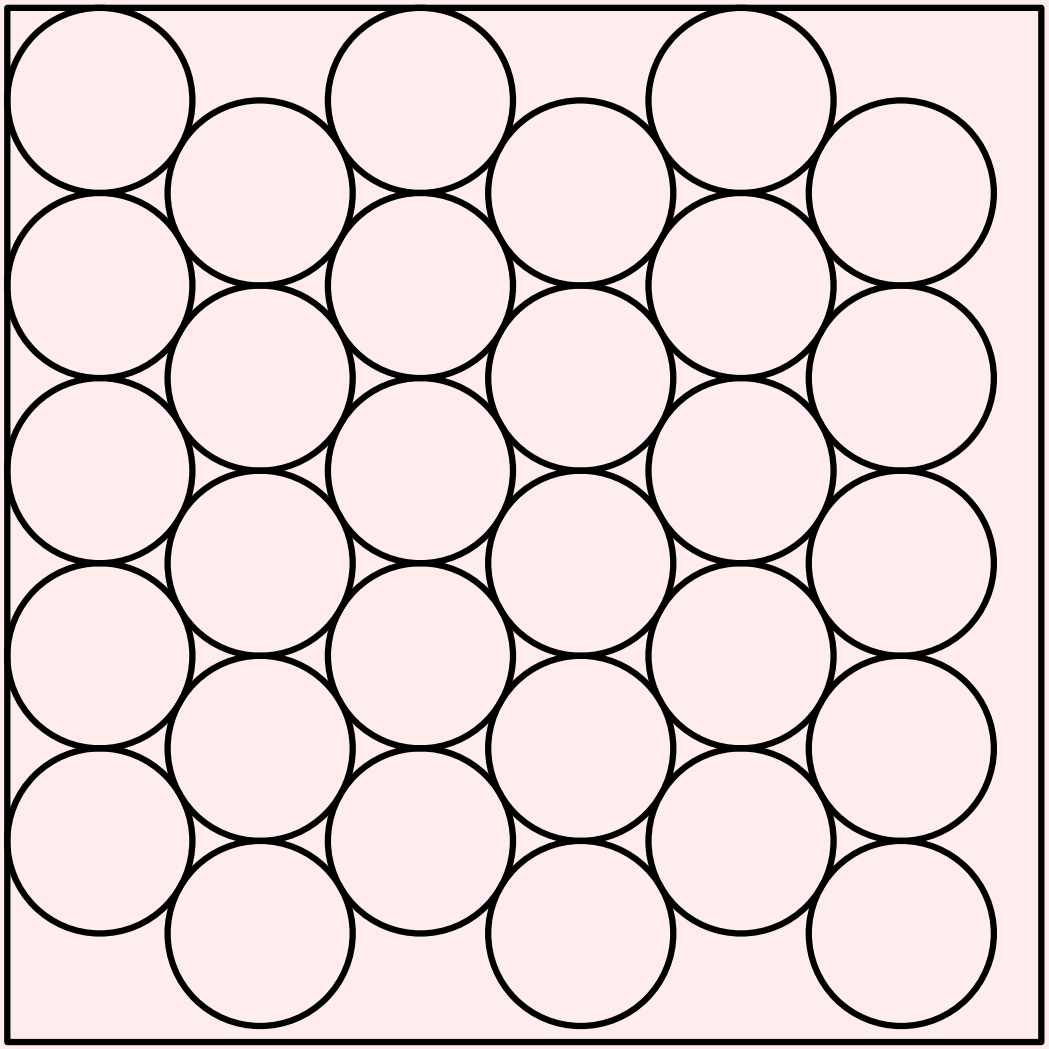
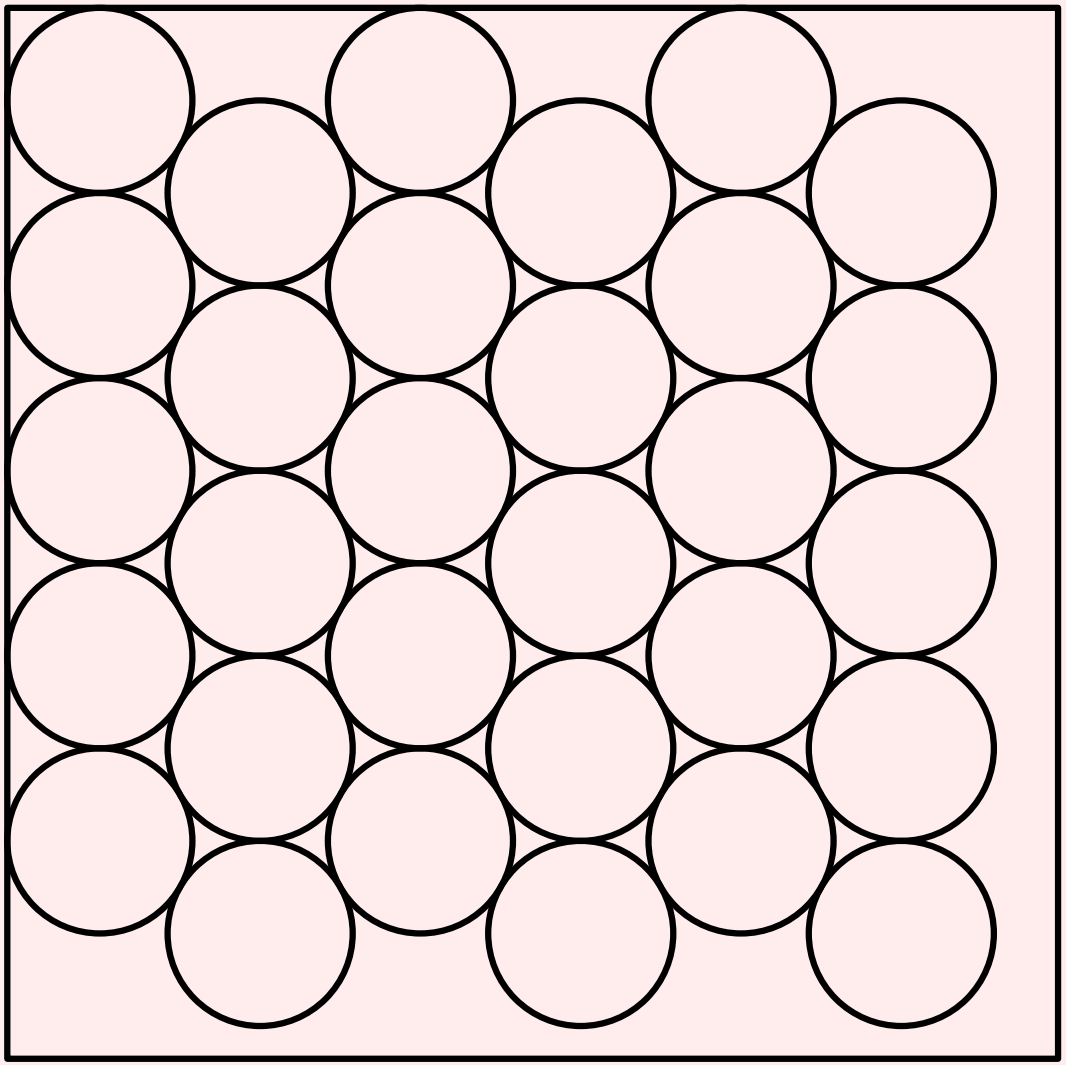


Simulation



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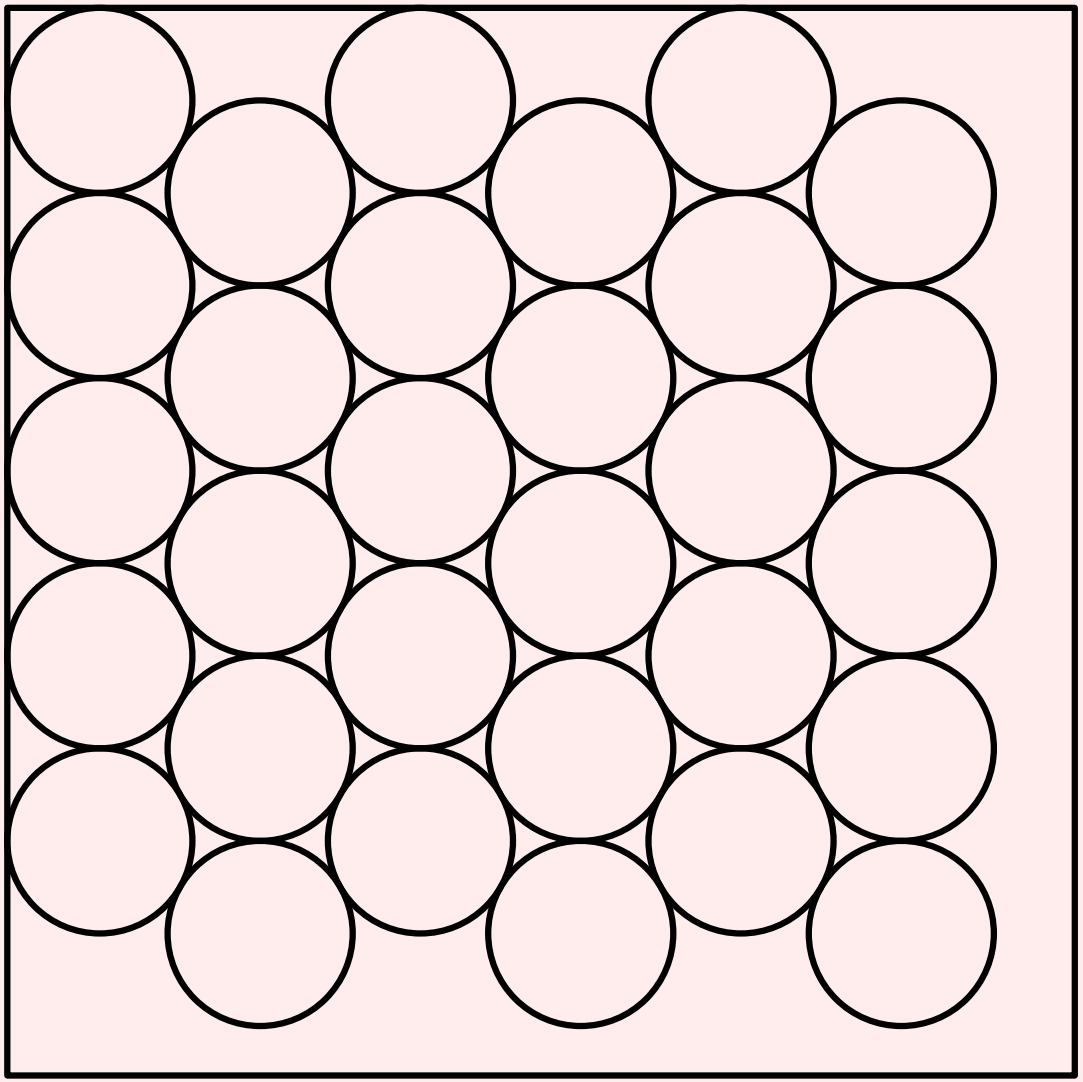
Simulation



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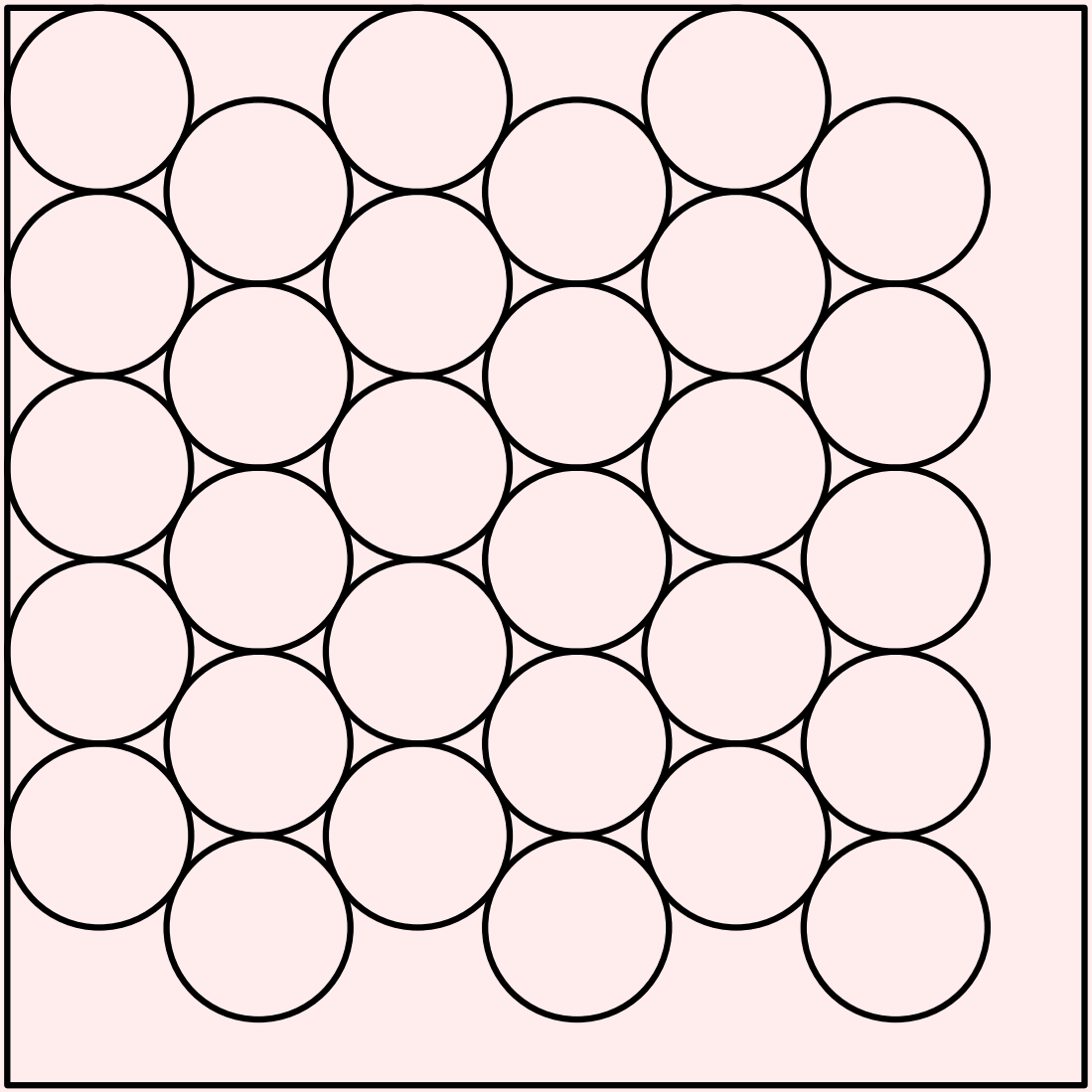

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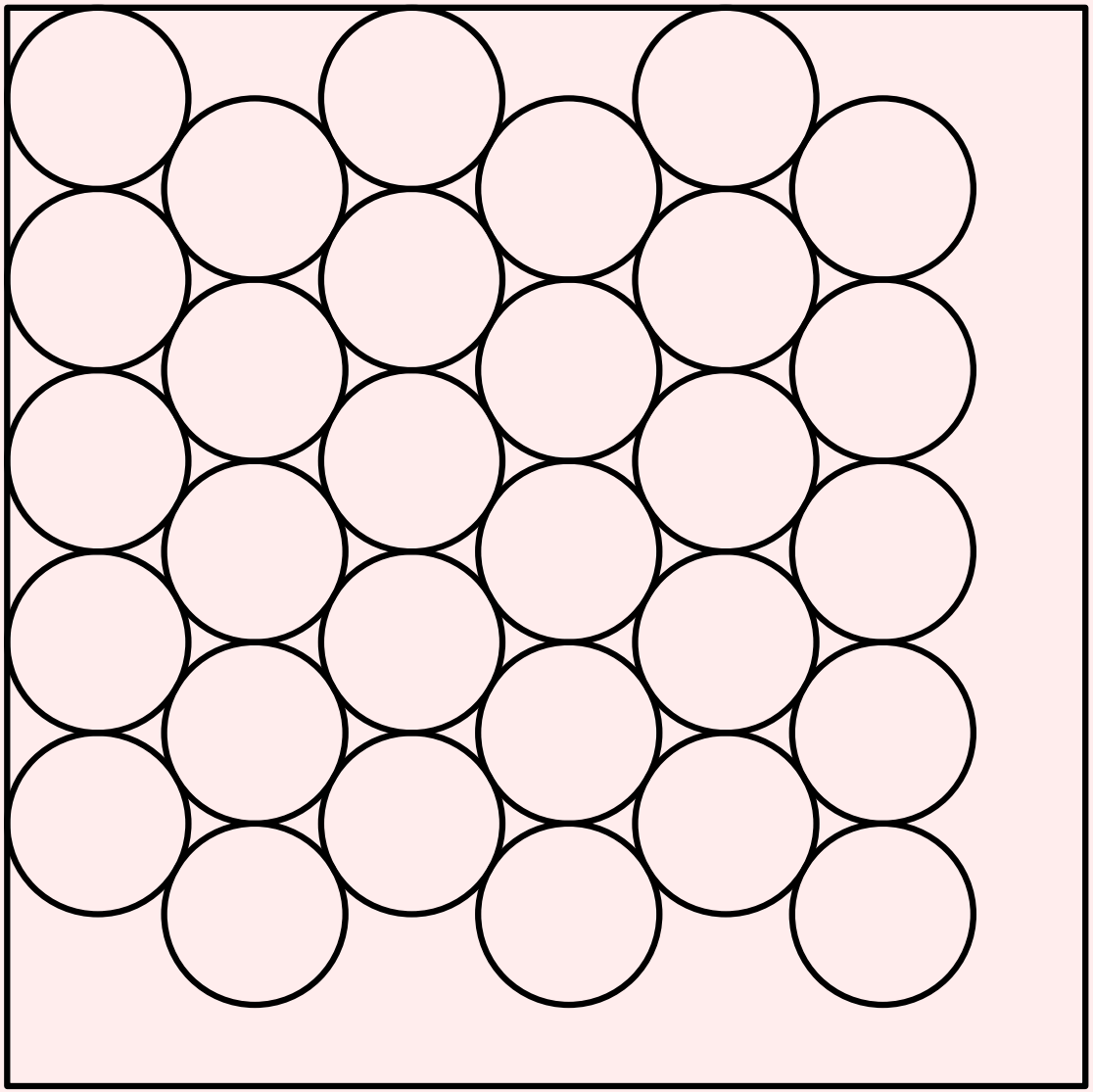

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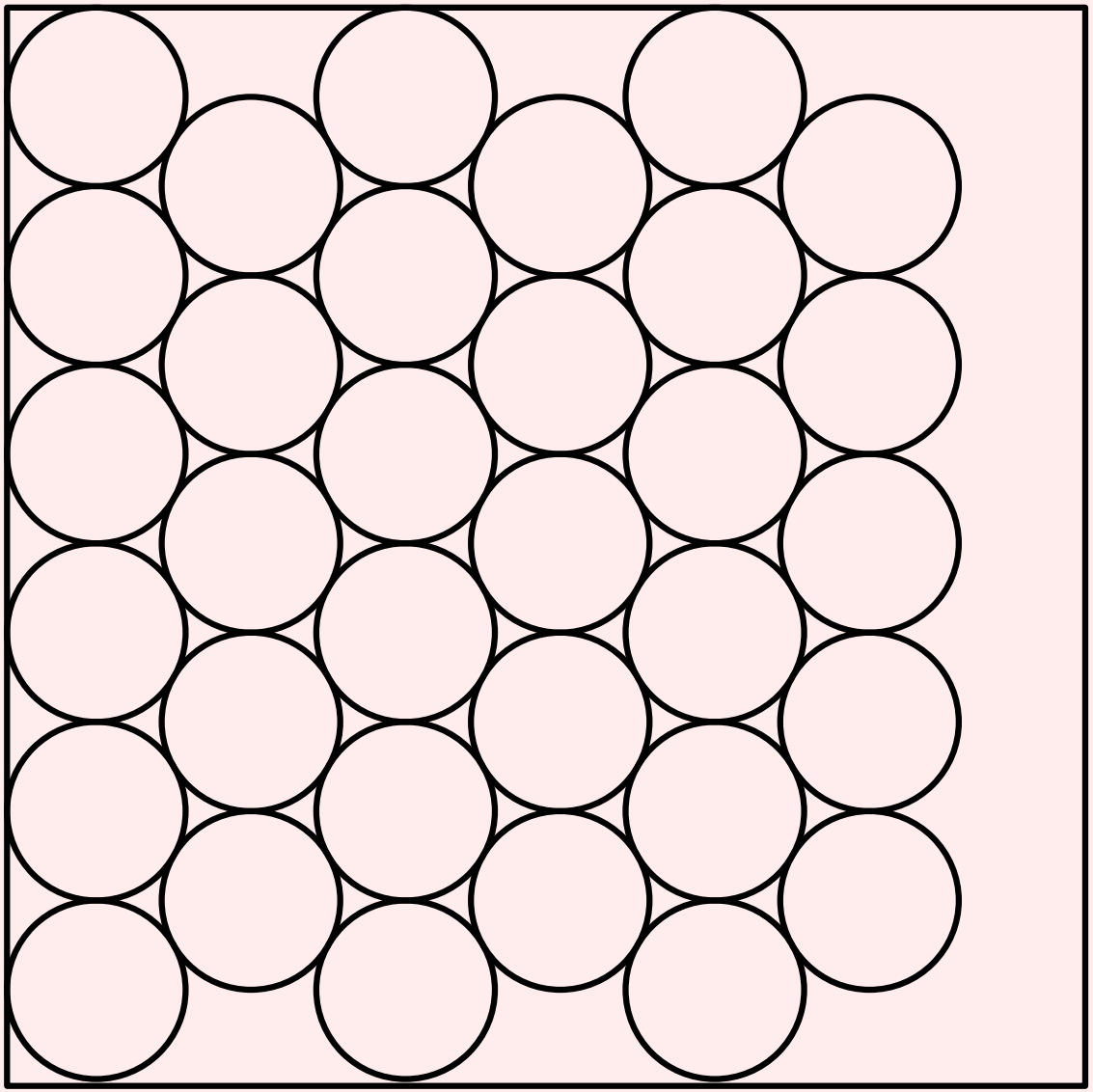

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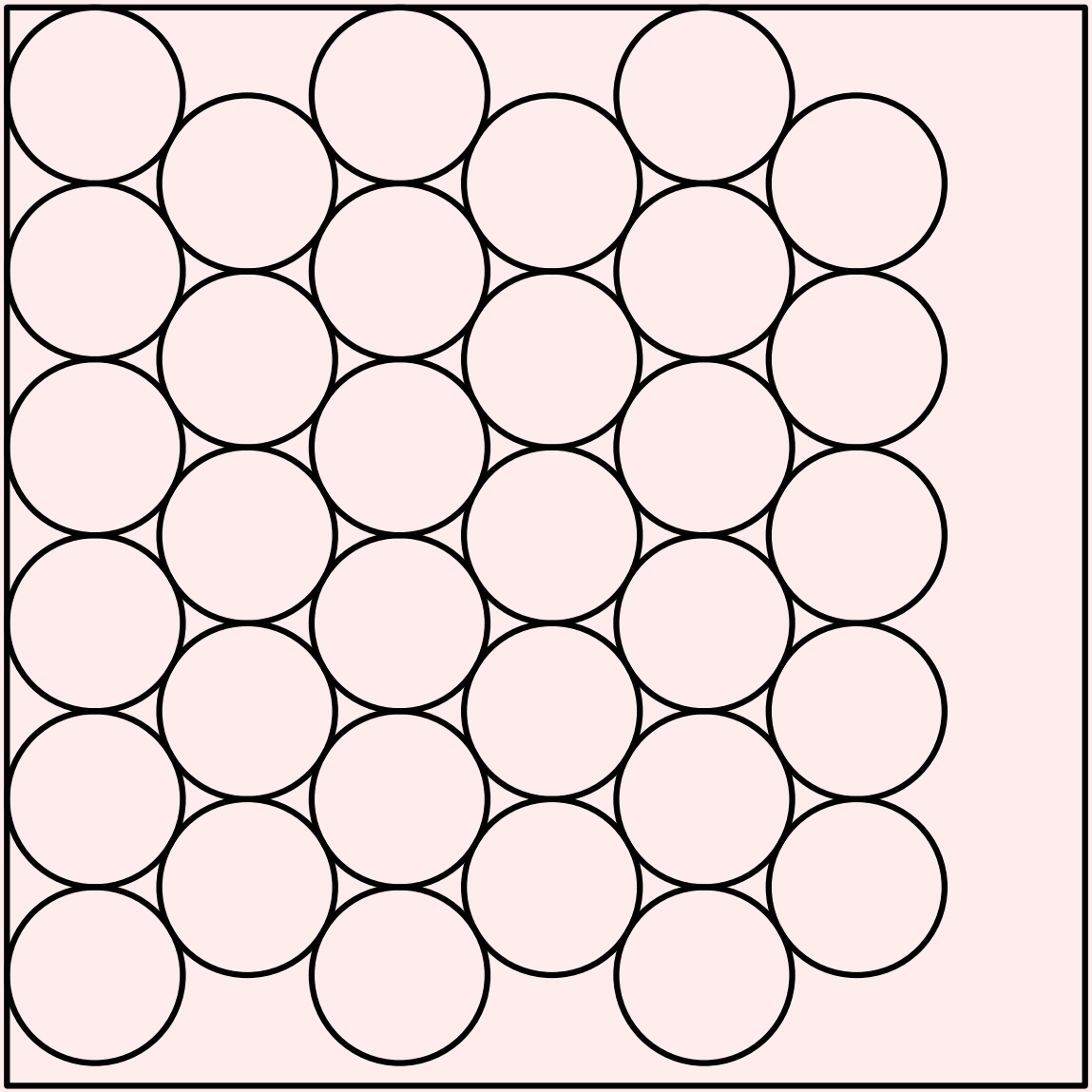
Simulation



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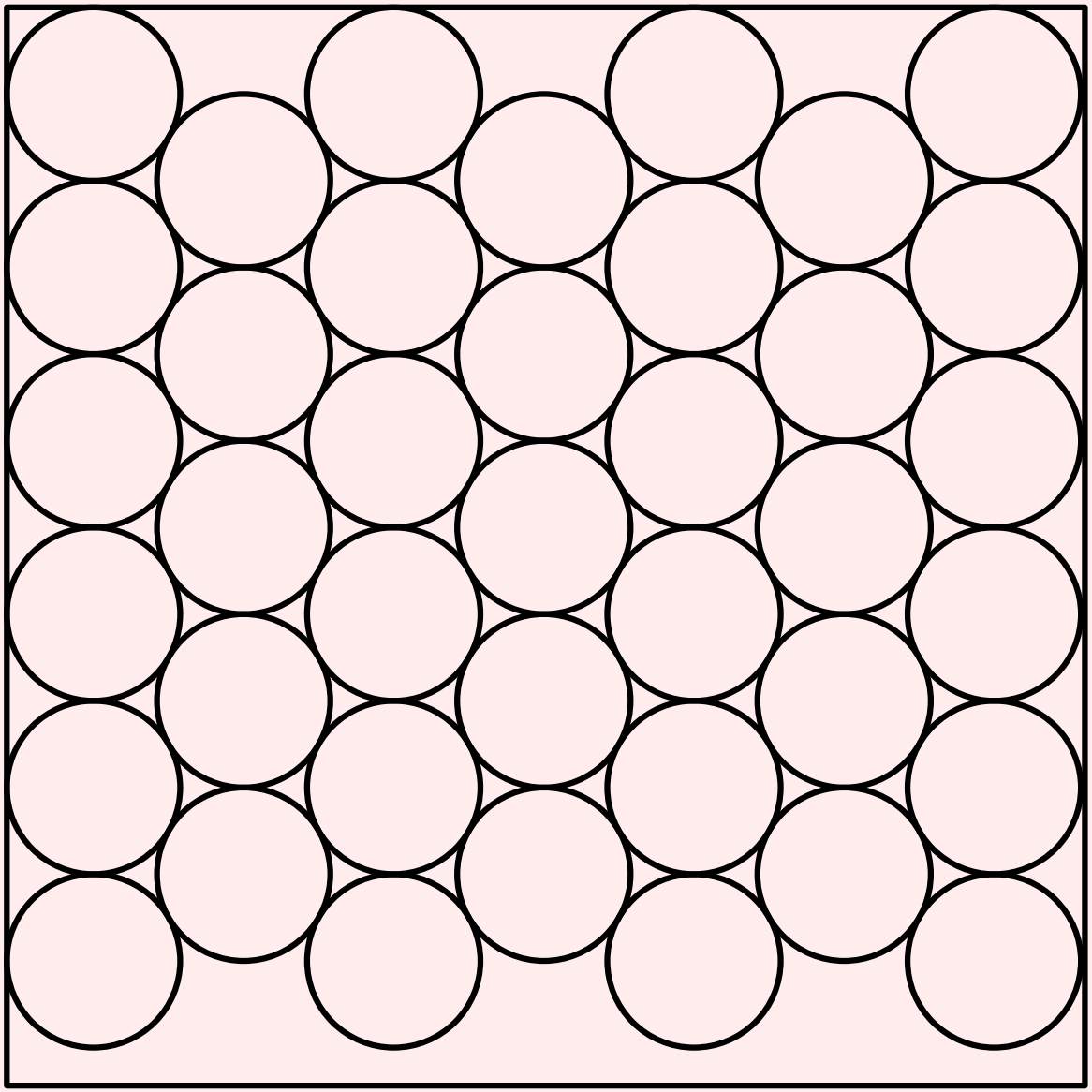
Simulation



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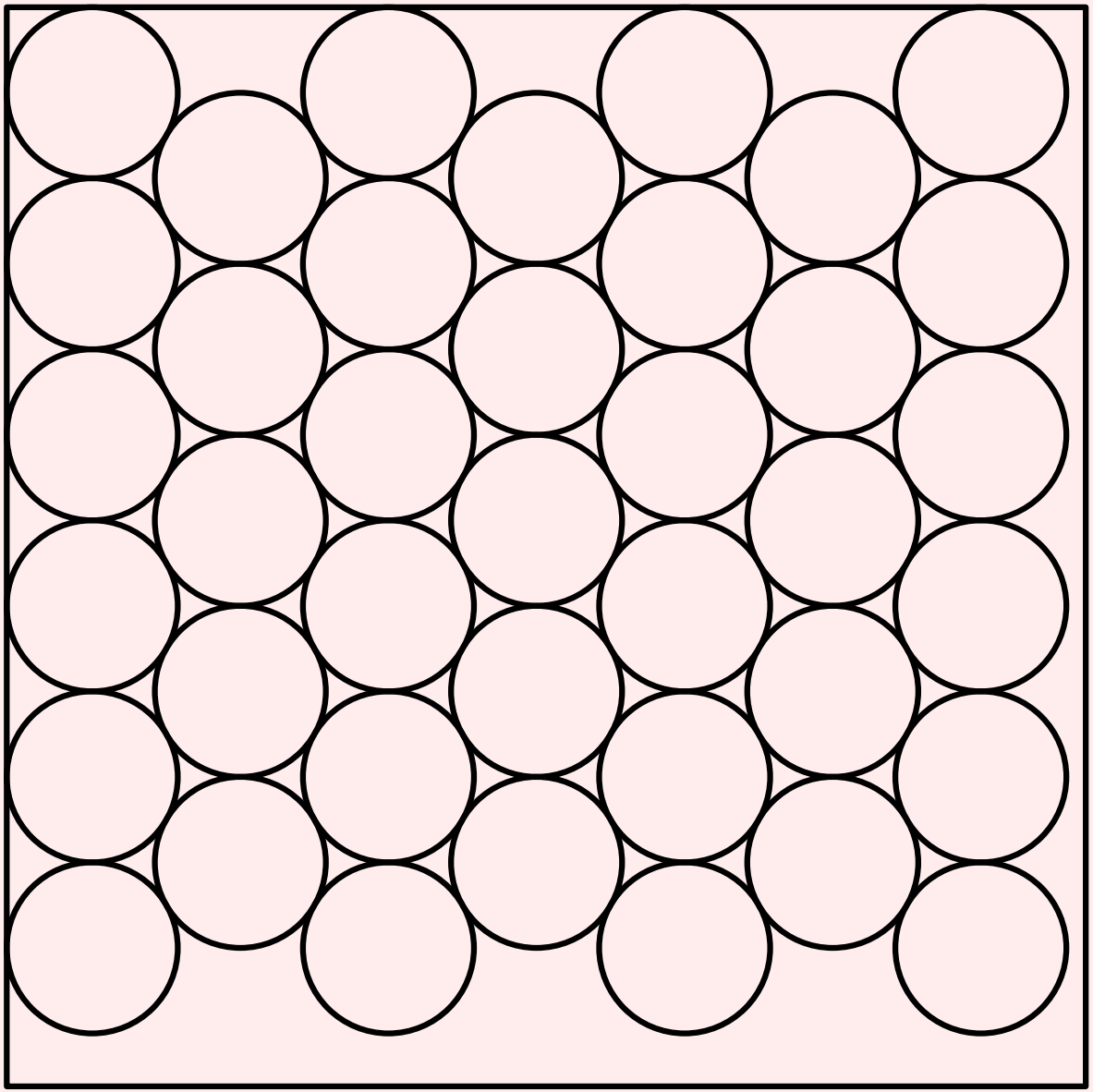

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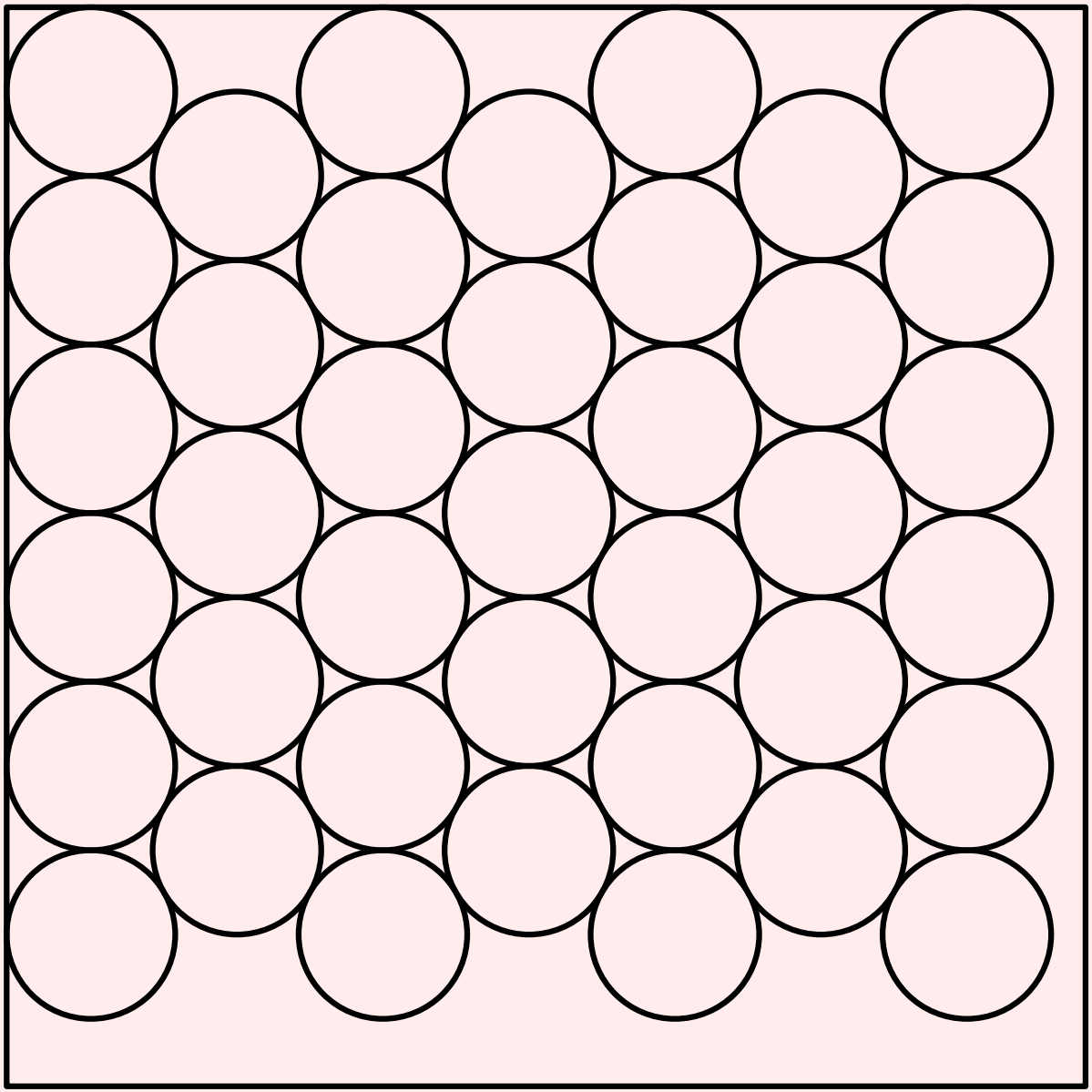
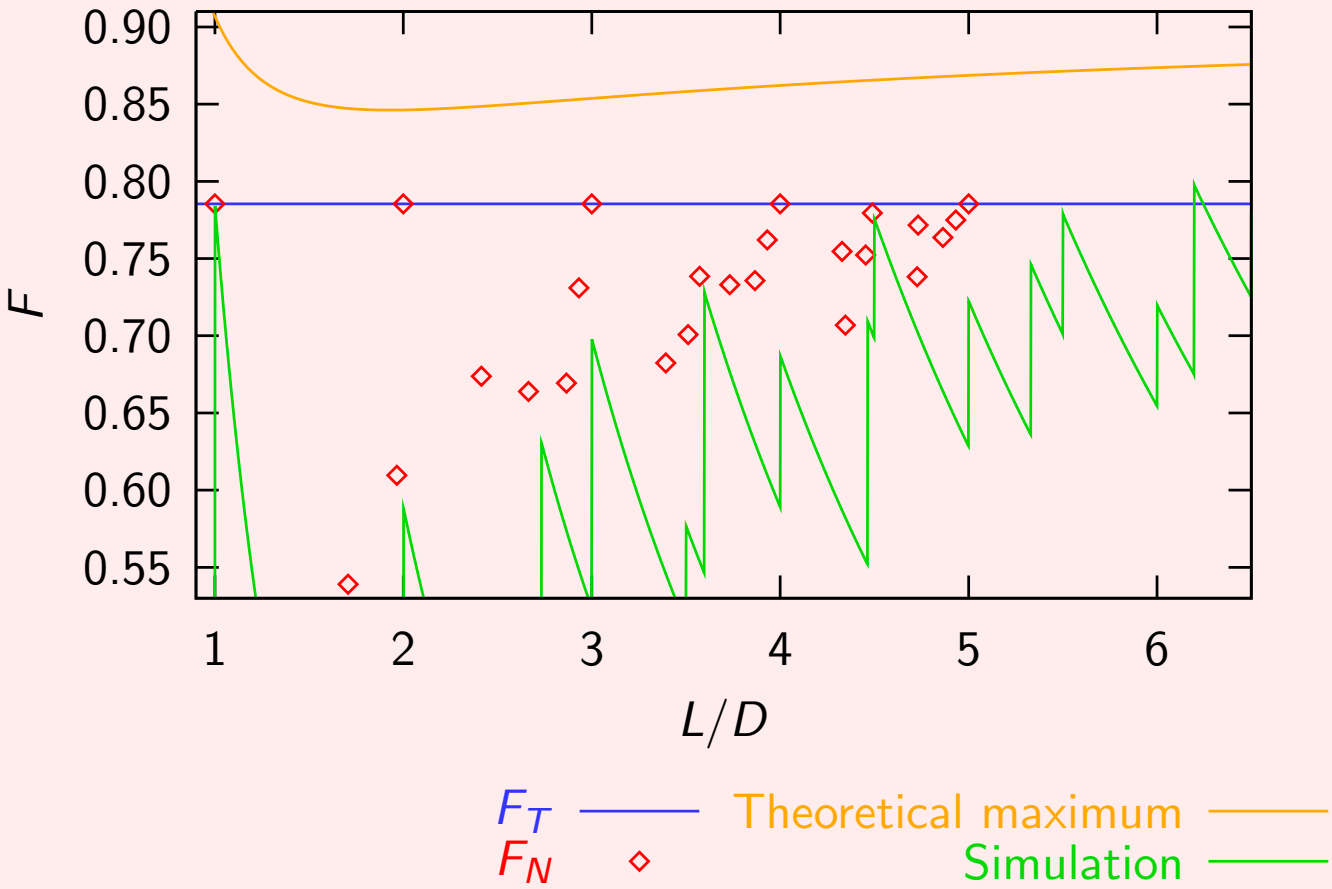
Simulation



Graph



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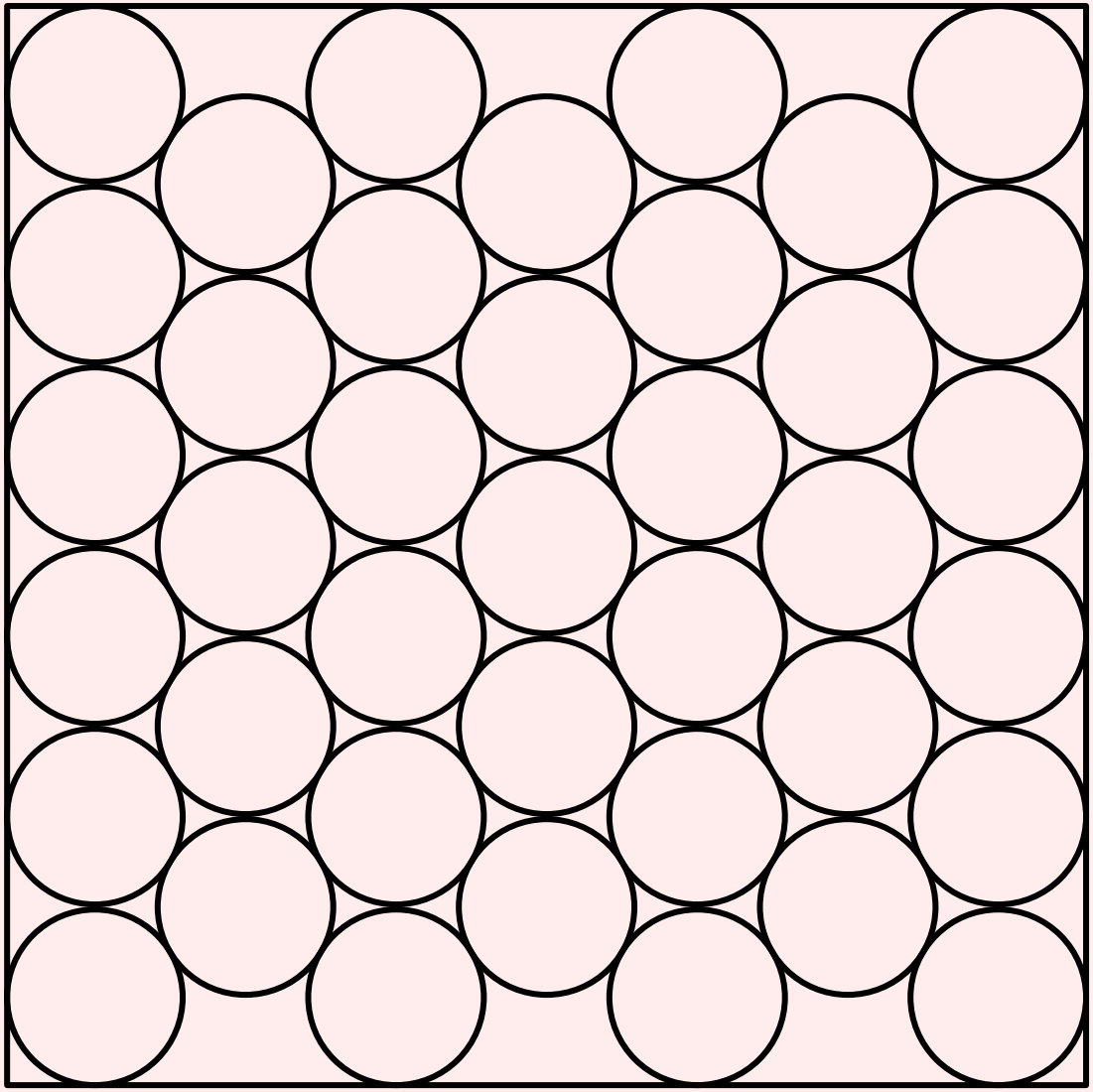
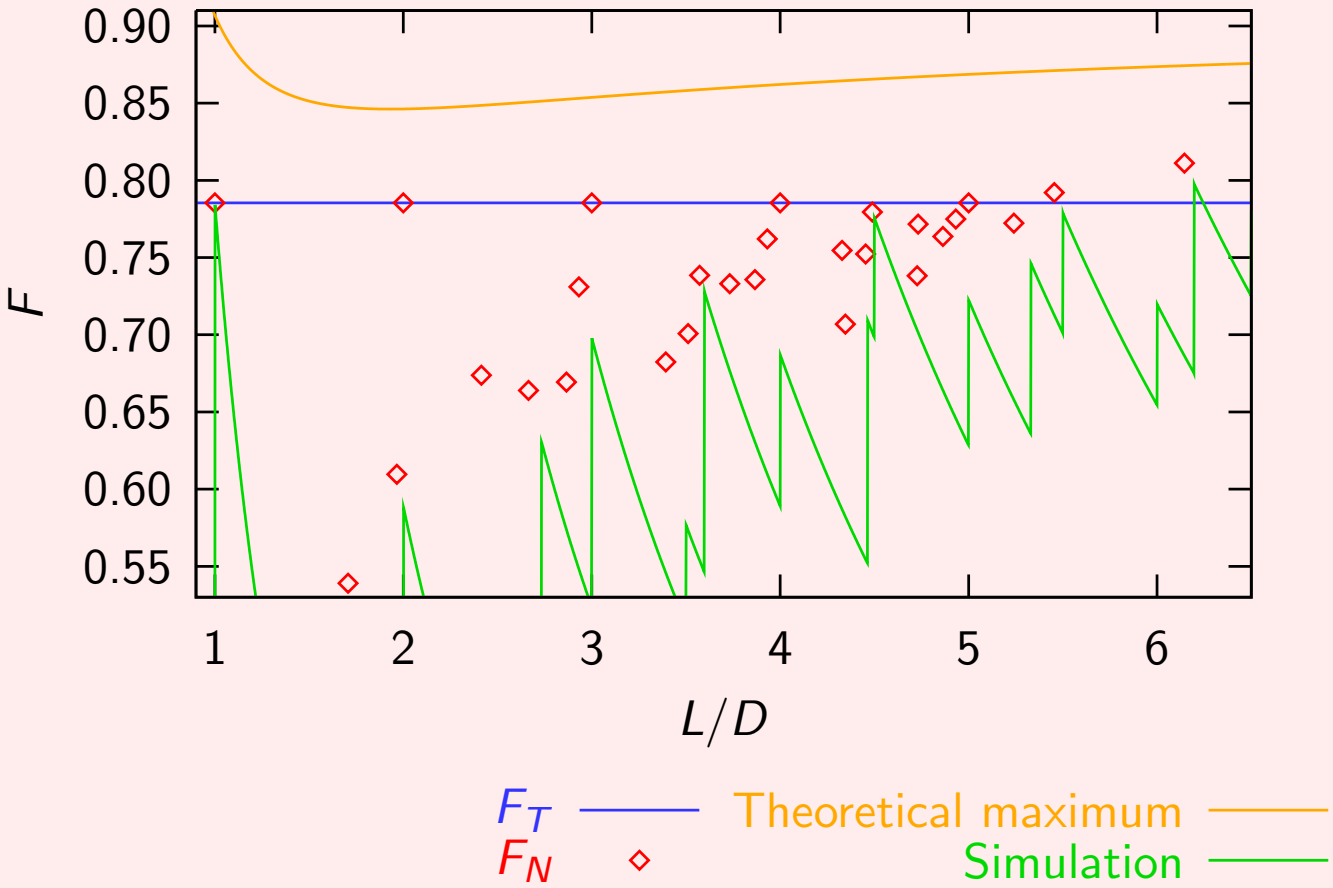
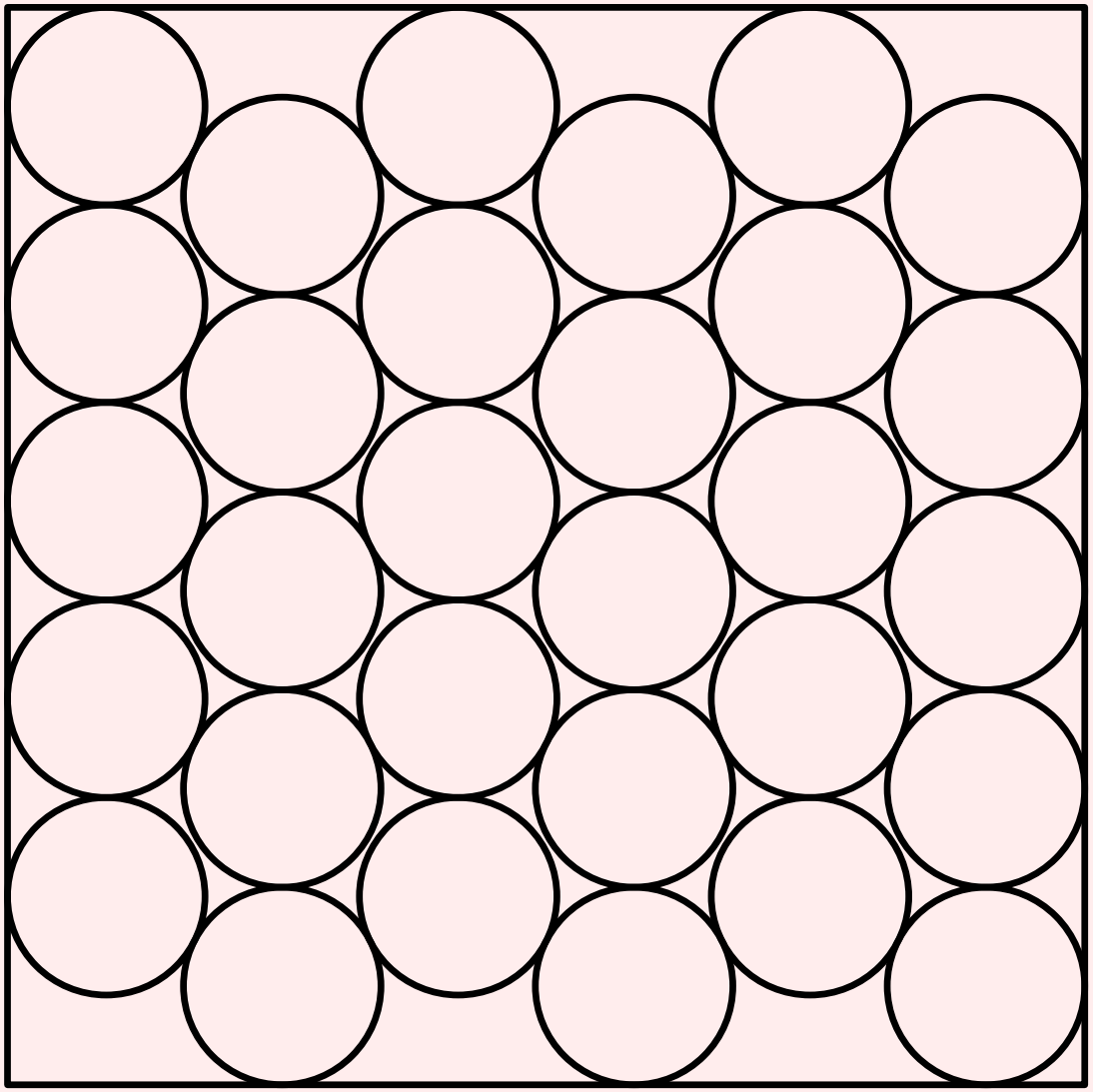


$$F_{39} \approx 0{,}811$$

Optimize



30

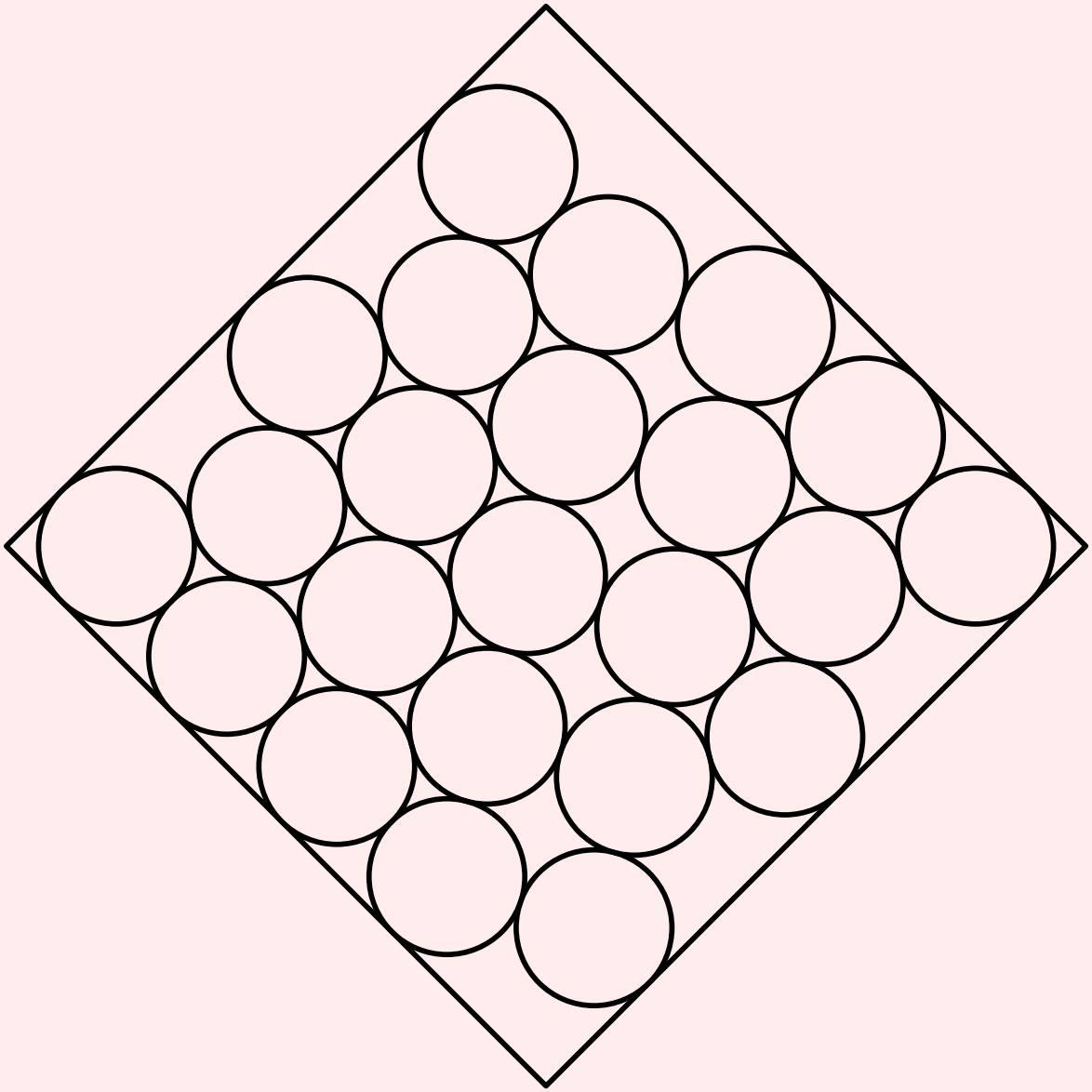


$$F_{30} \approx 0{,}792$$

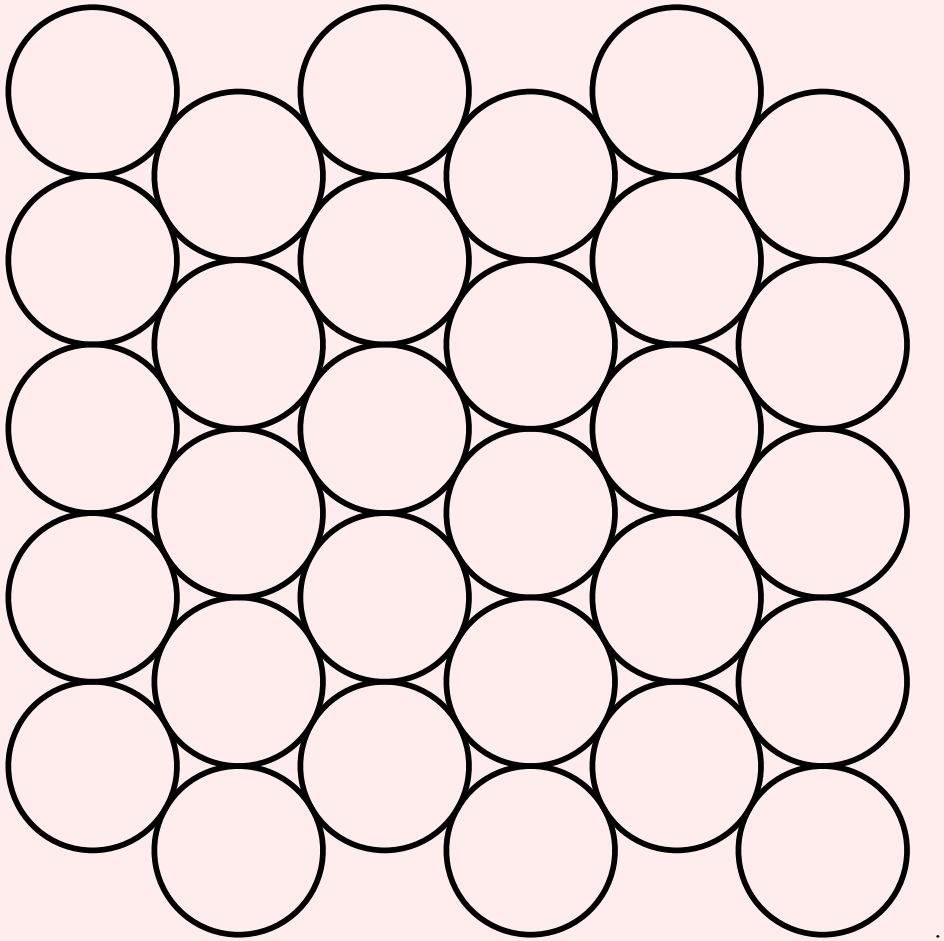
(SM)

- What's the use of this?

Go beyond



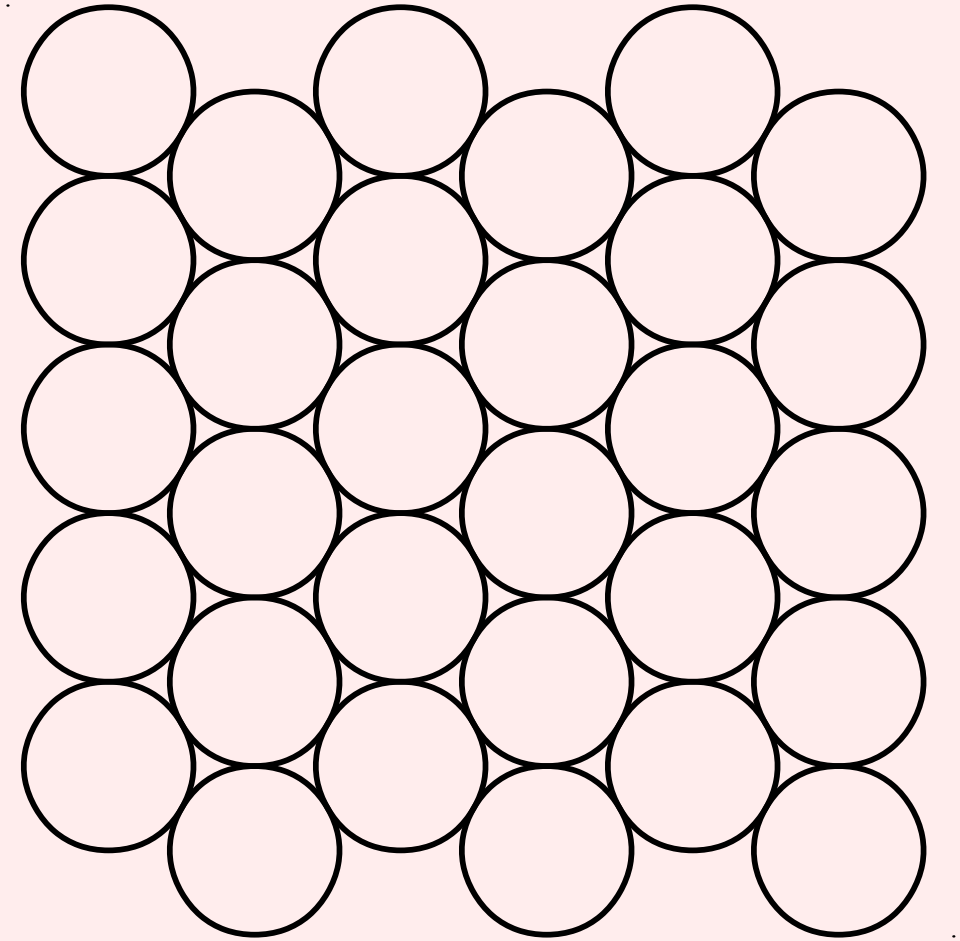
2D bubbles



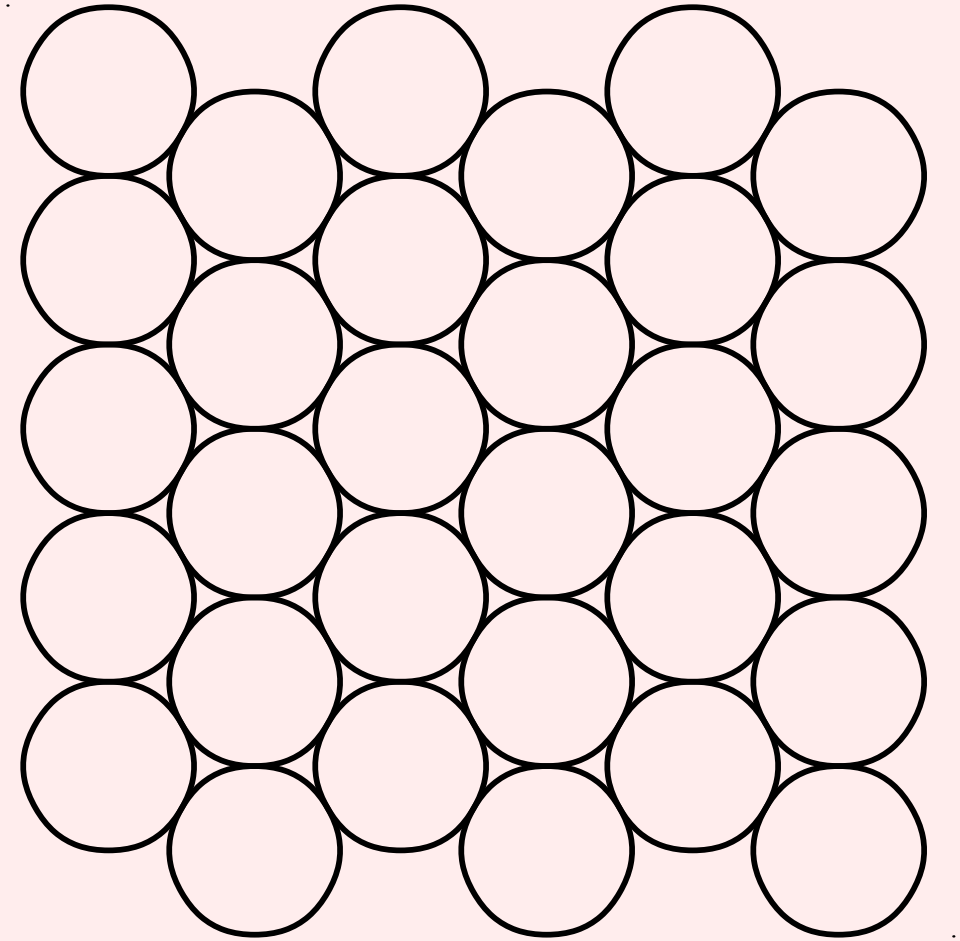
What is the smallest number of circles that can cover more area of a square than a single circle? An example of the scientific method in action.

Luís Nobre Gonçalves

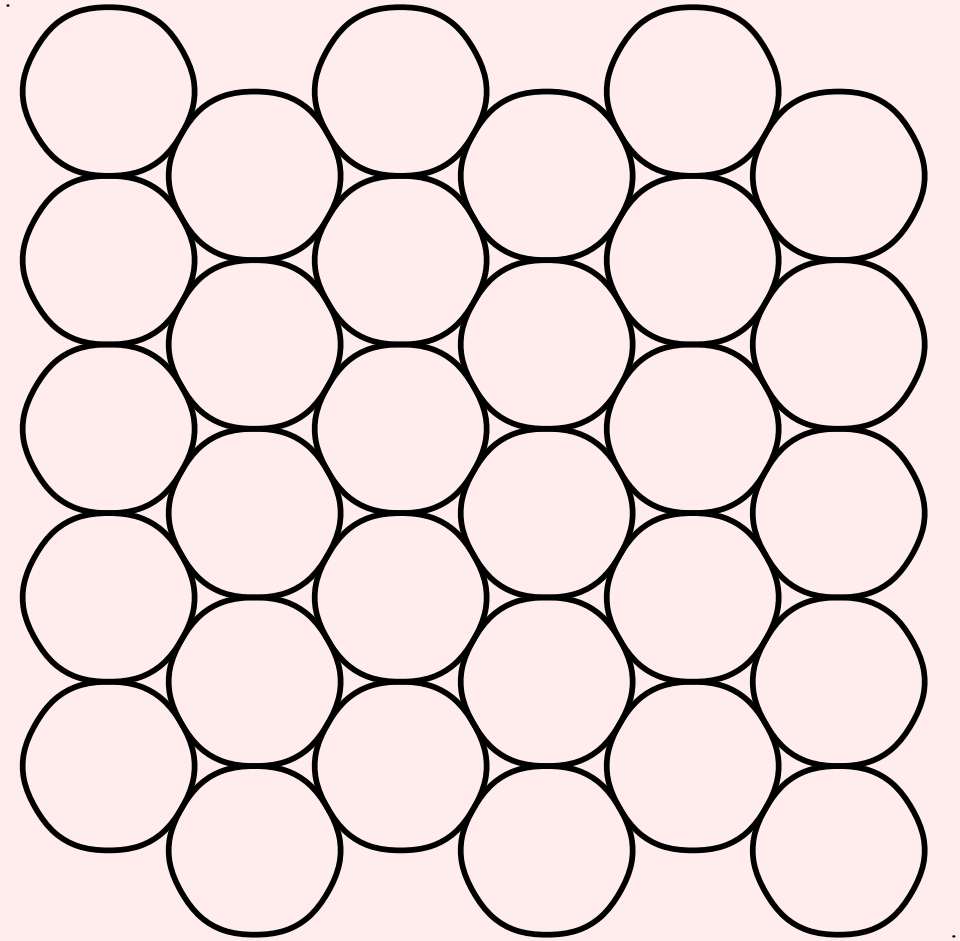
2D bubbles



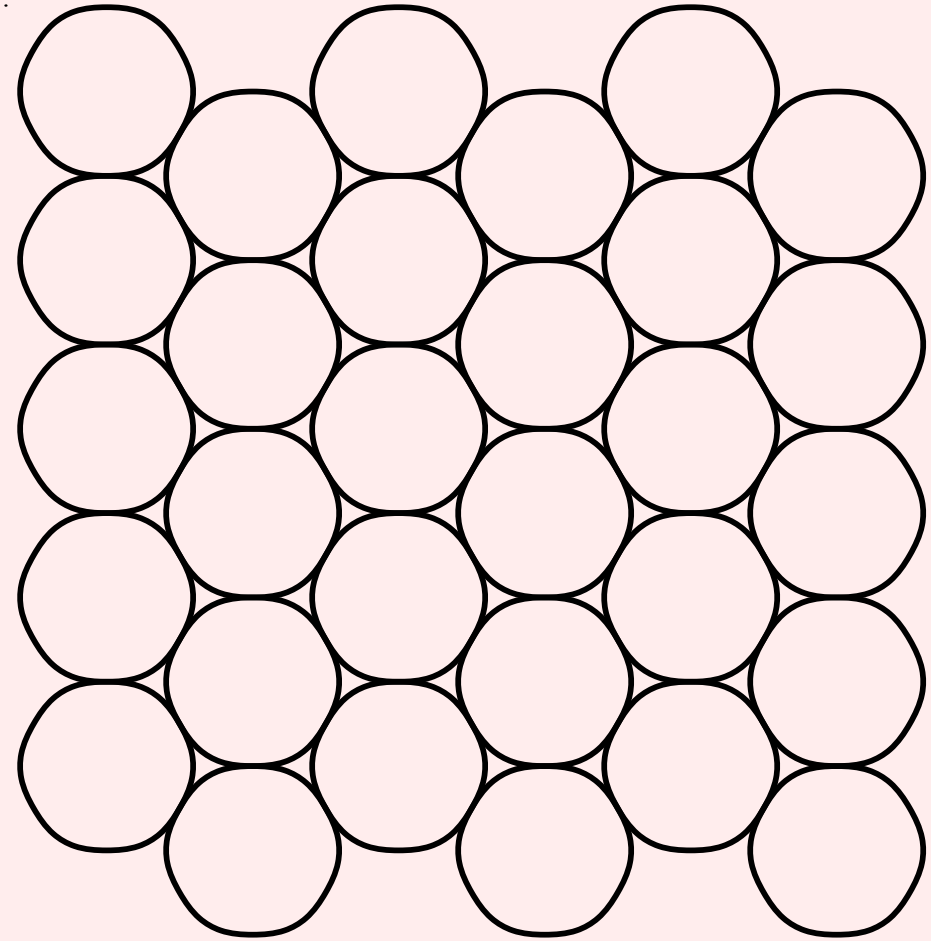
2D bubbles



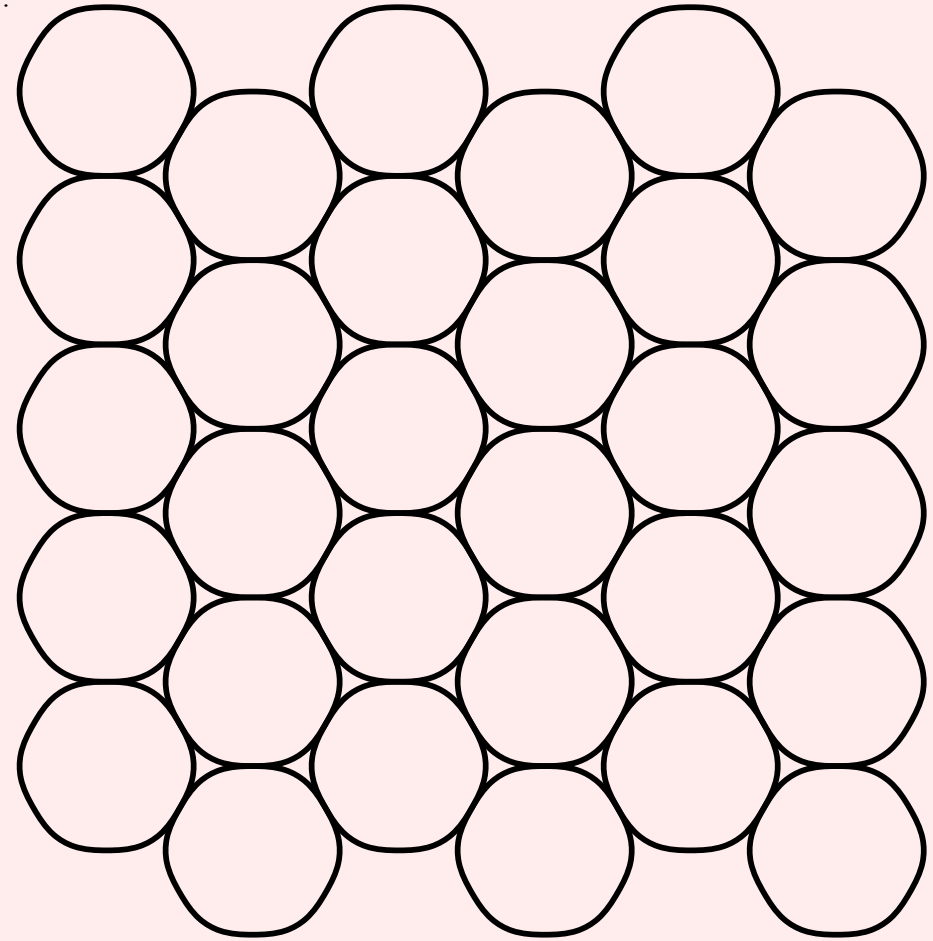
2D bubbles



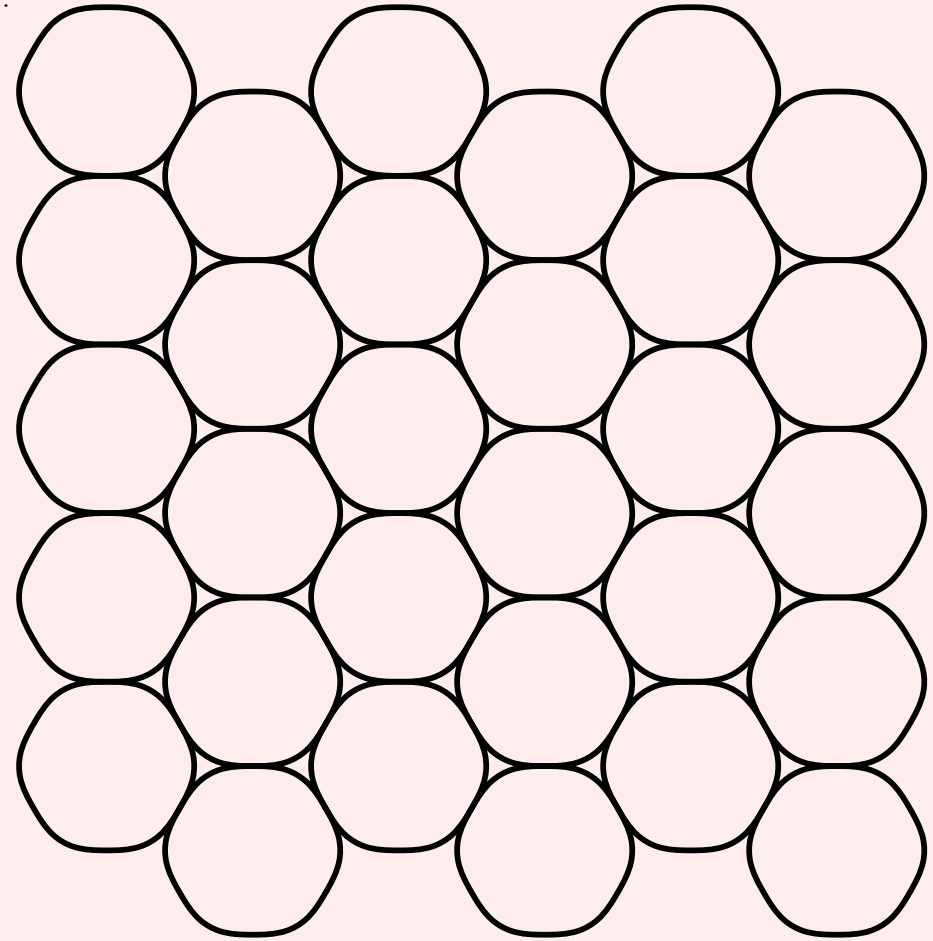
2D bubbles



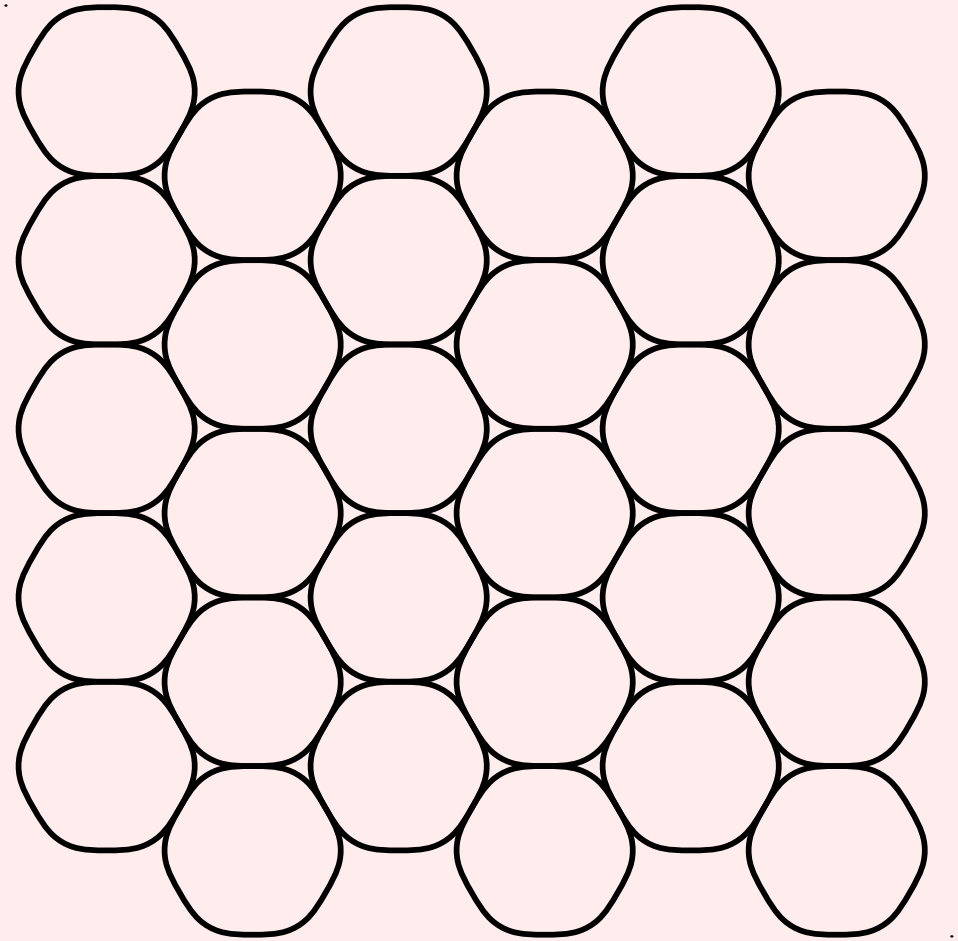
2D bubbles



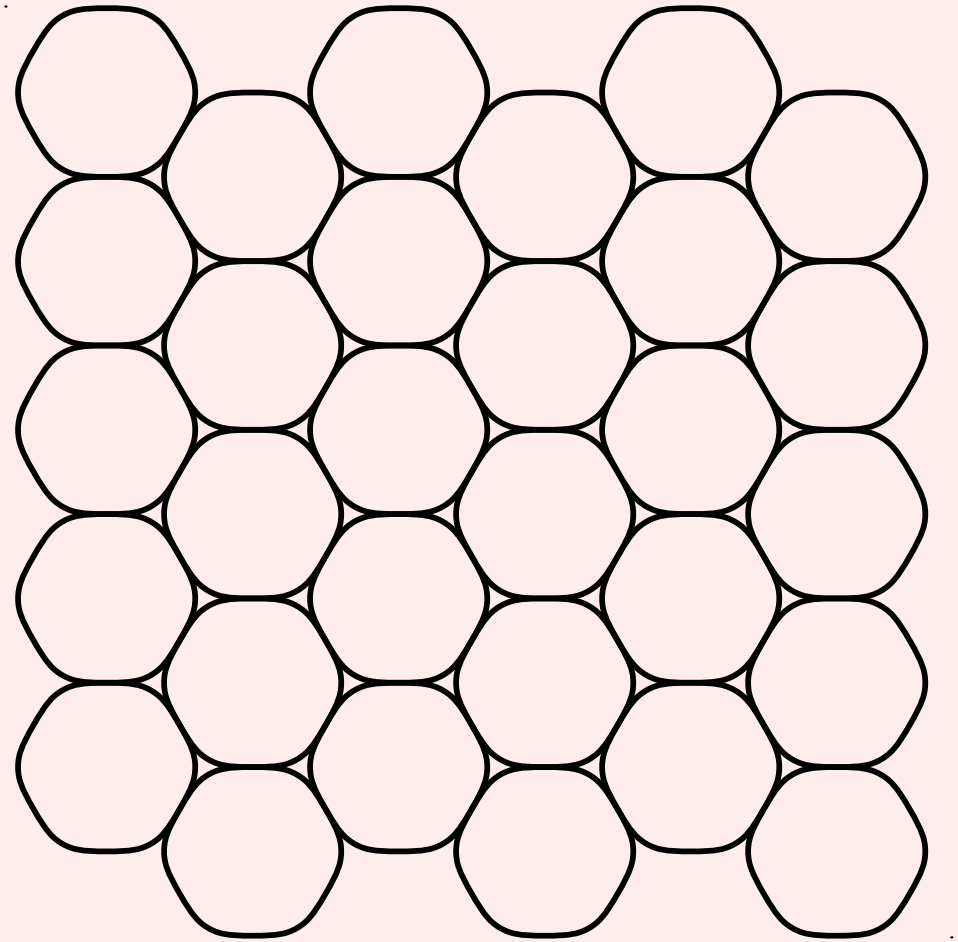
2D bubbles



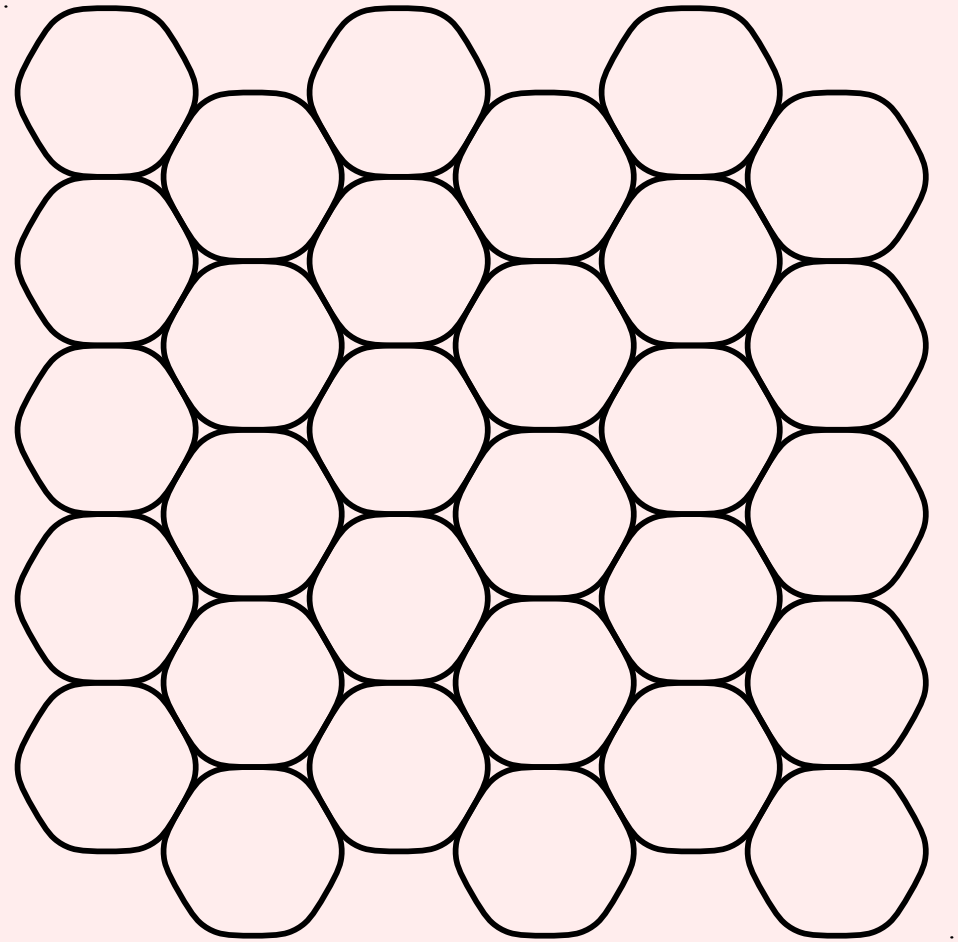
2D bubbles



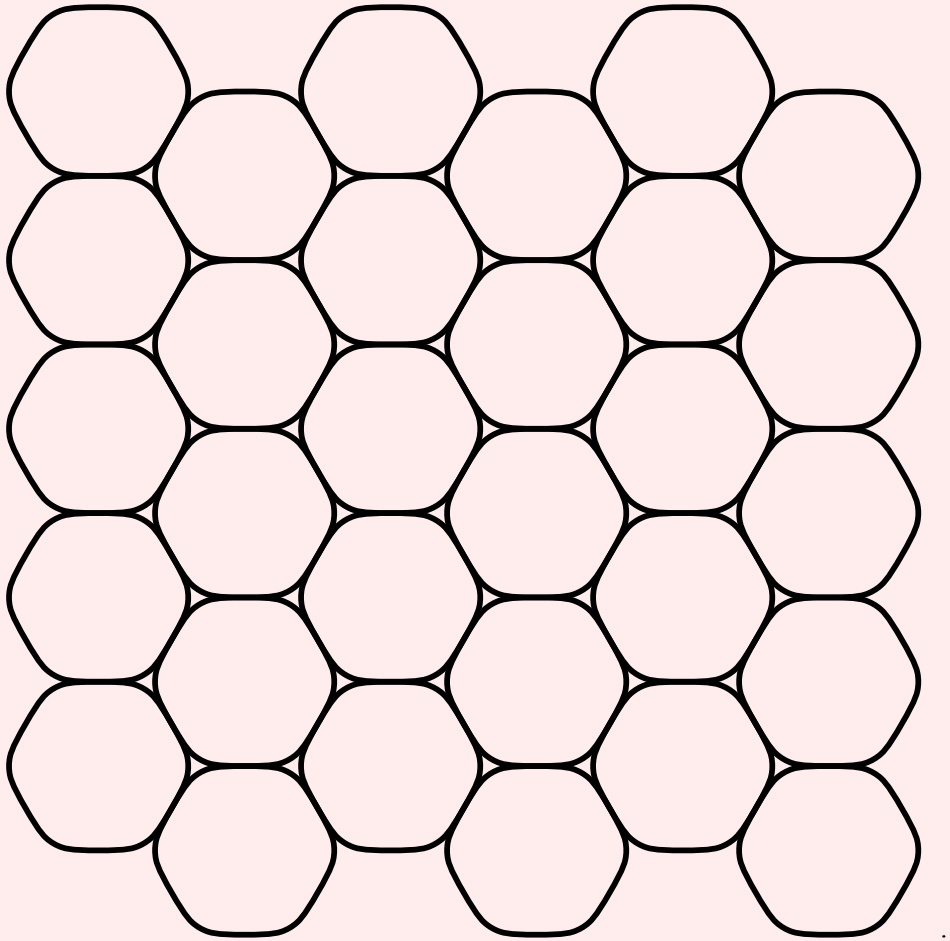
2D bubbles



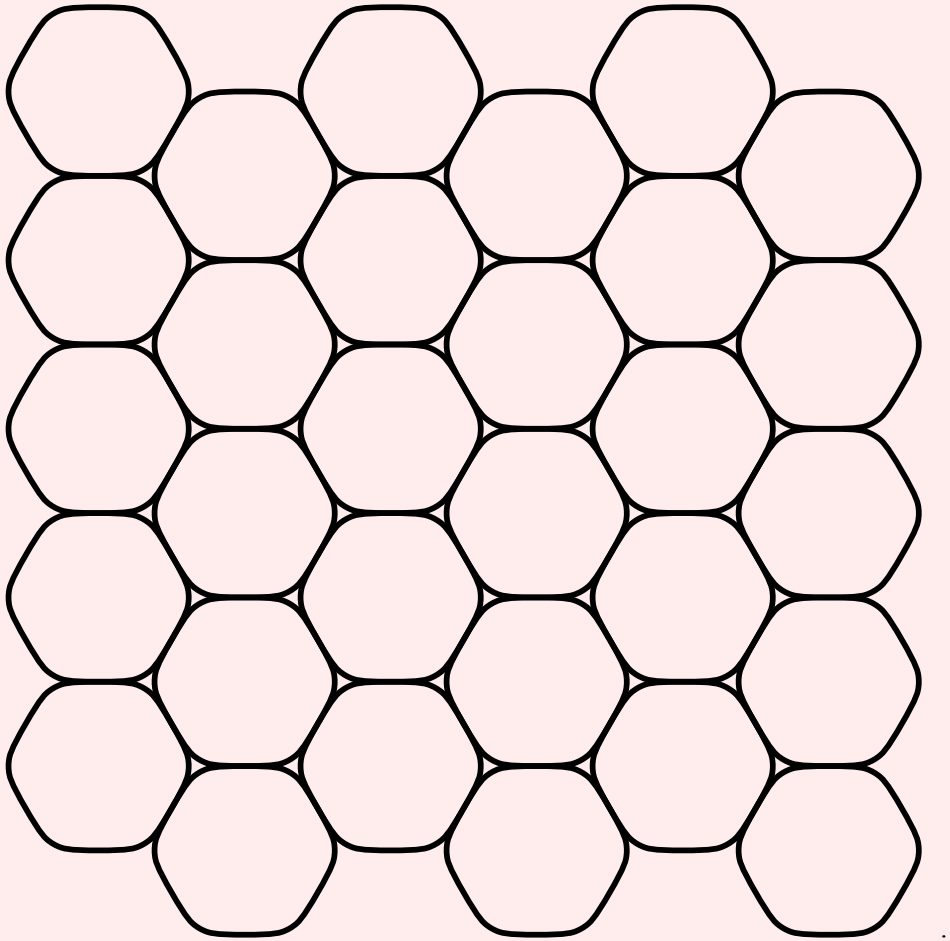
2D bubbles



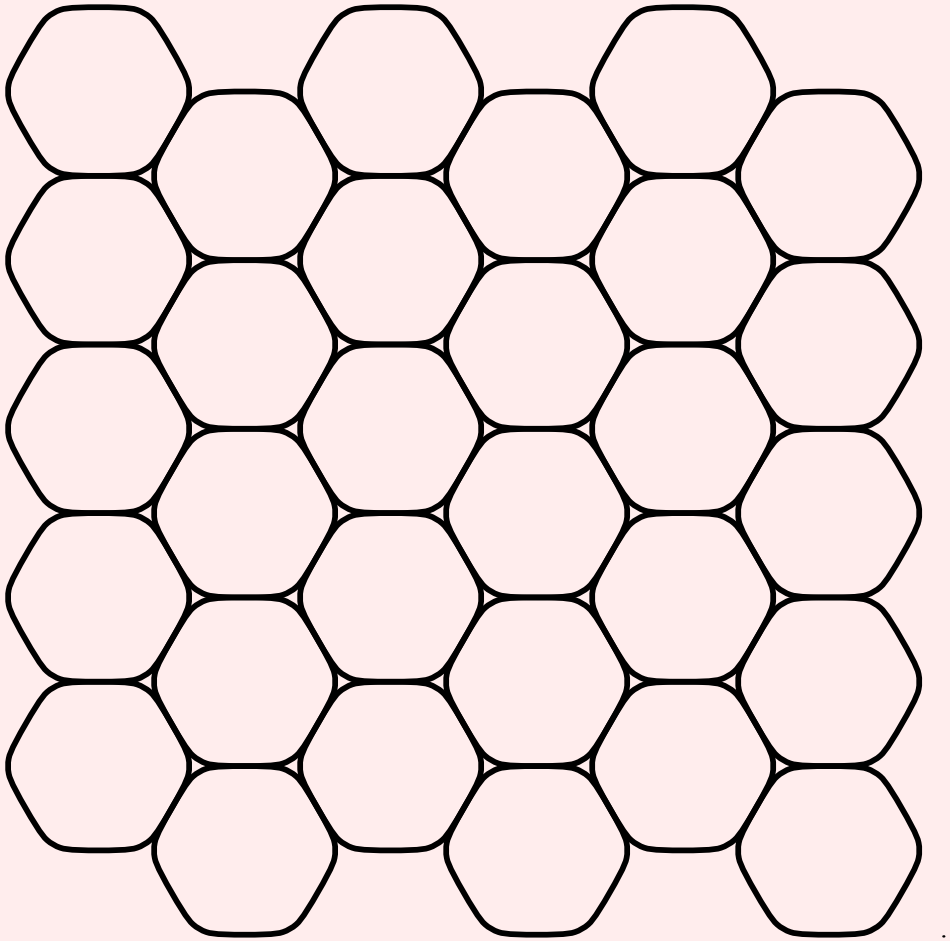
2D bubbles



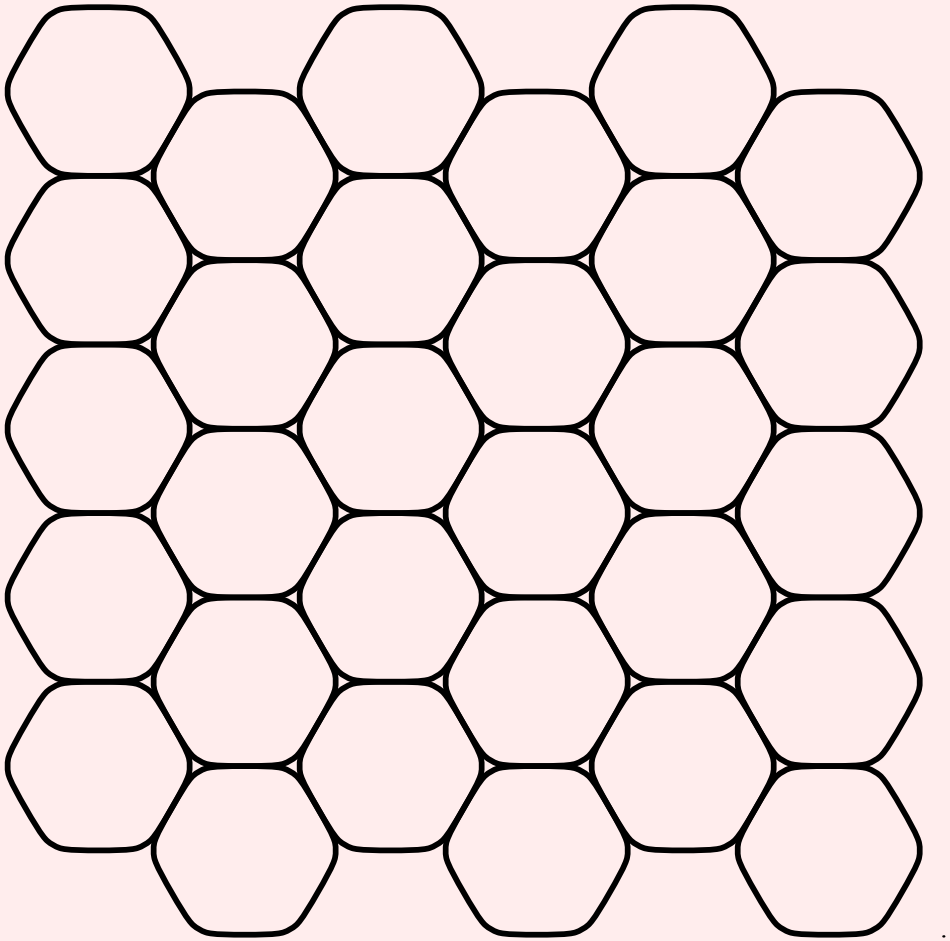
2D bubbles



2D bubbles



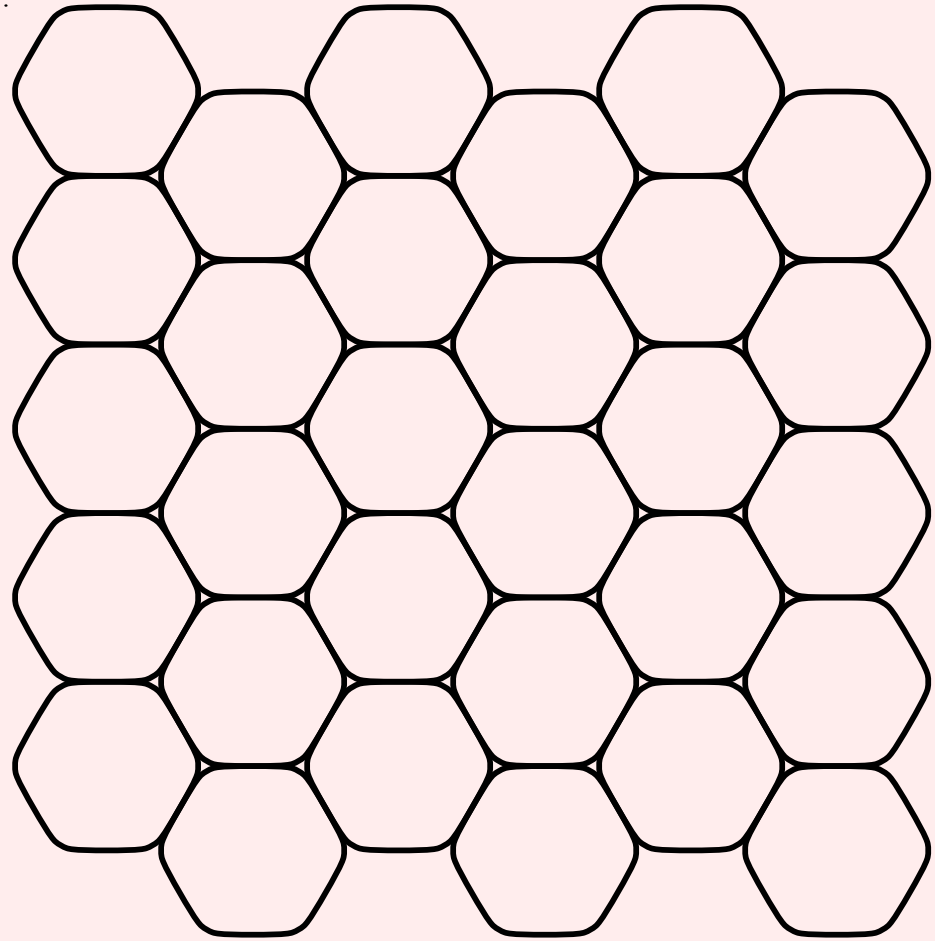
2D bubbles



What is the smallest number of circles that can cover more area of a square than a single circle? An example of the scientific method in action.

Luís Nobre Gonçalves

2D bubbles



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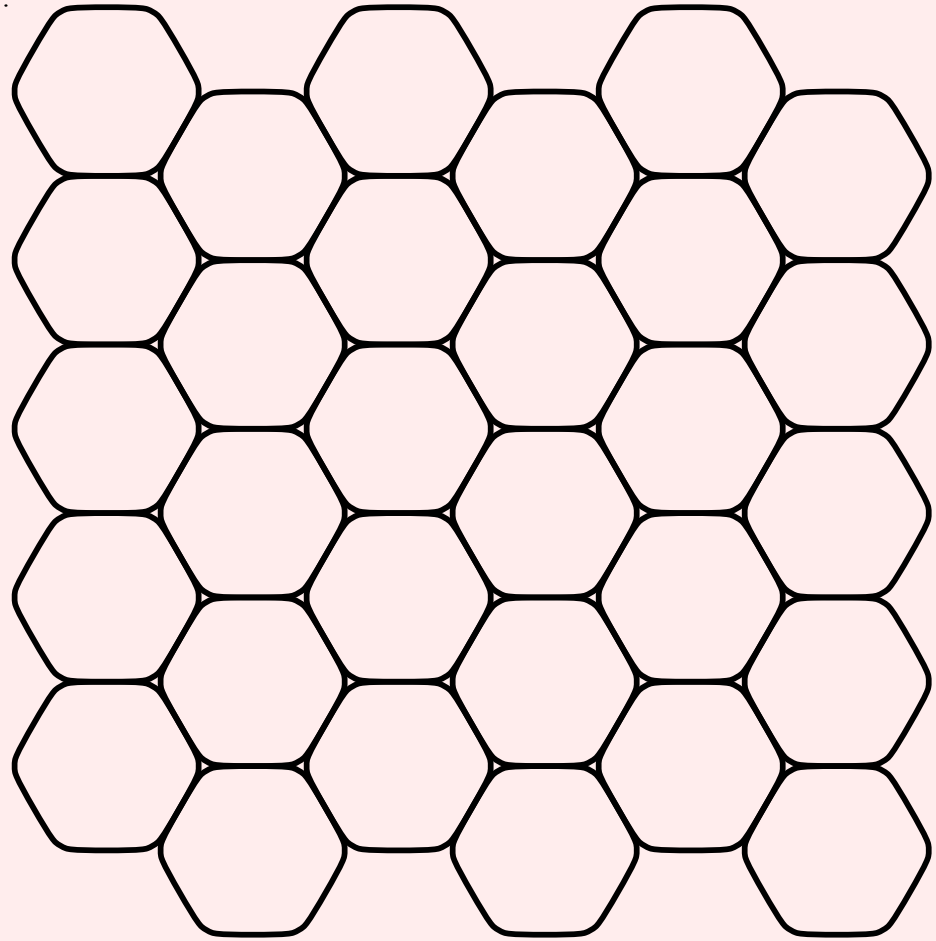
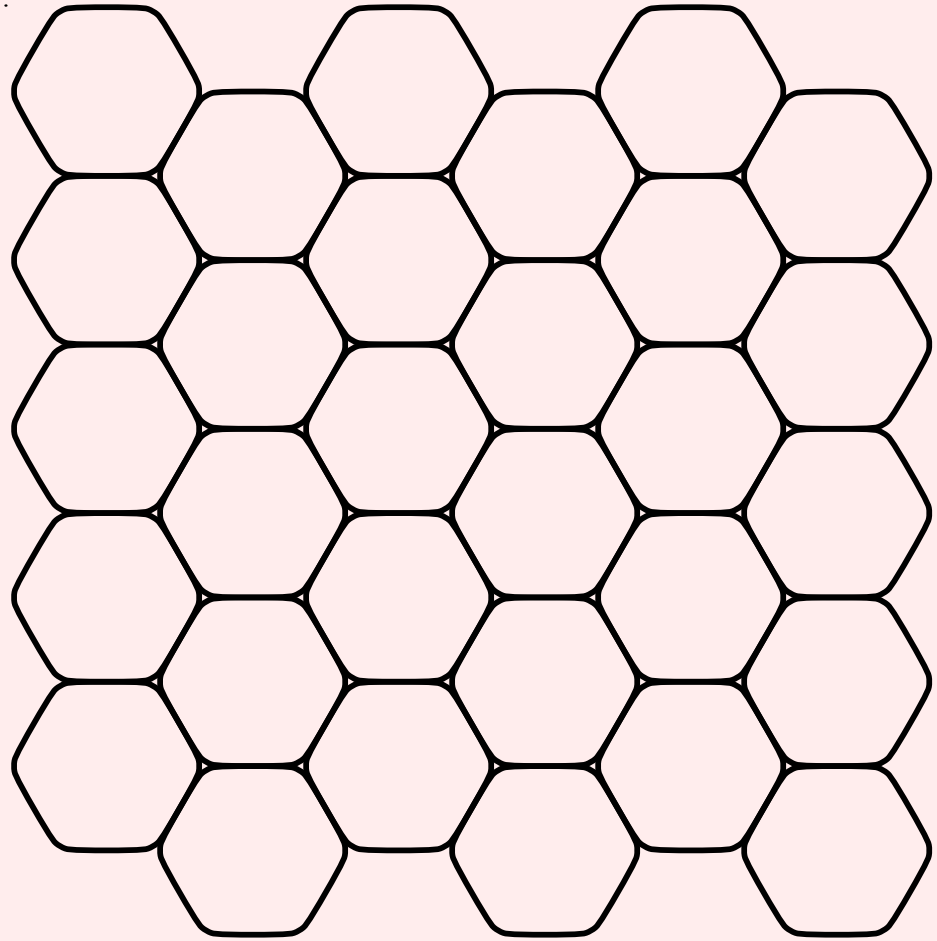
Luís Nobre Gonçalves

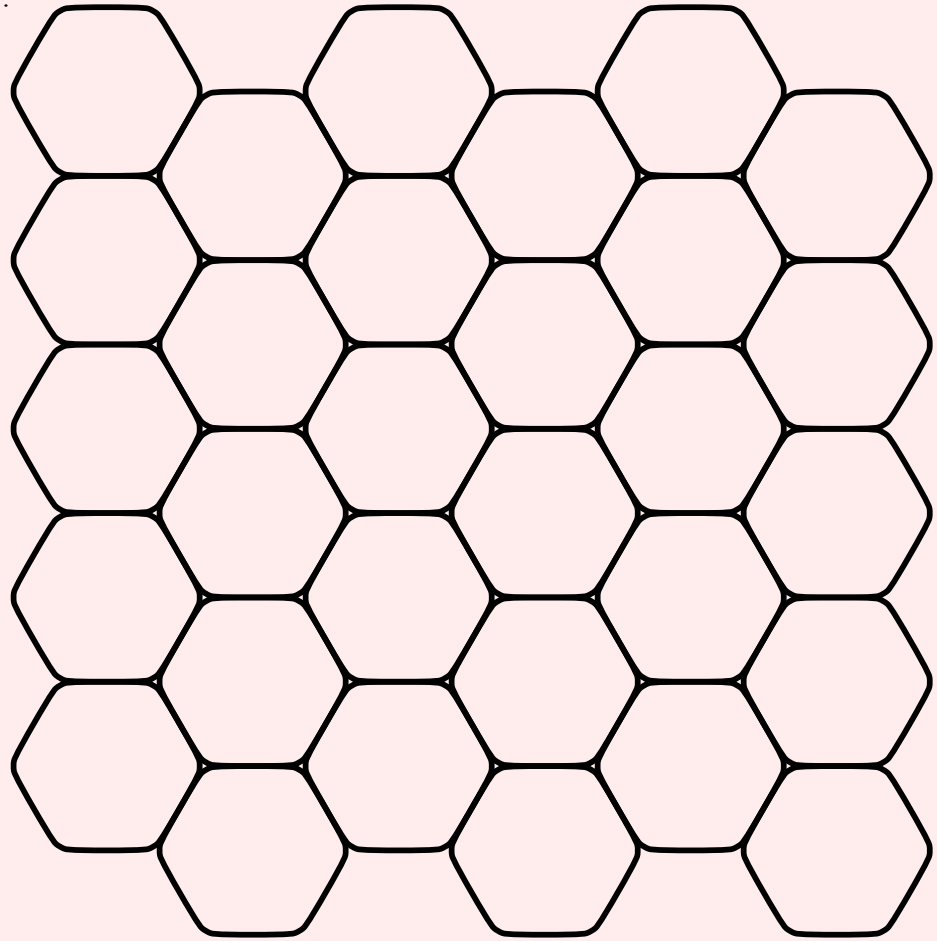
2D bubbles



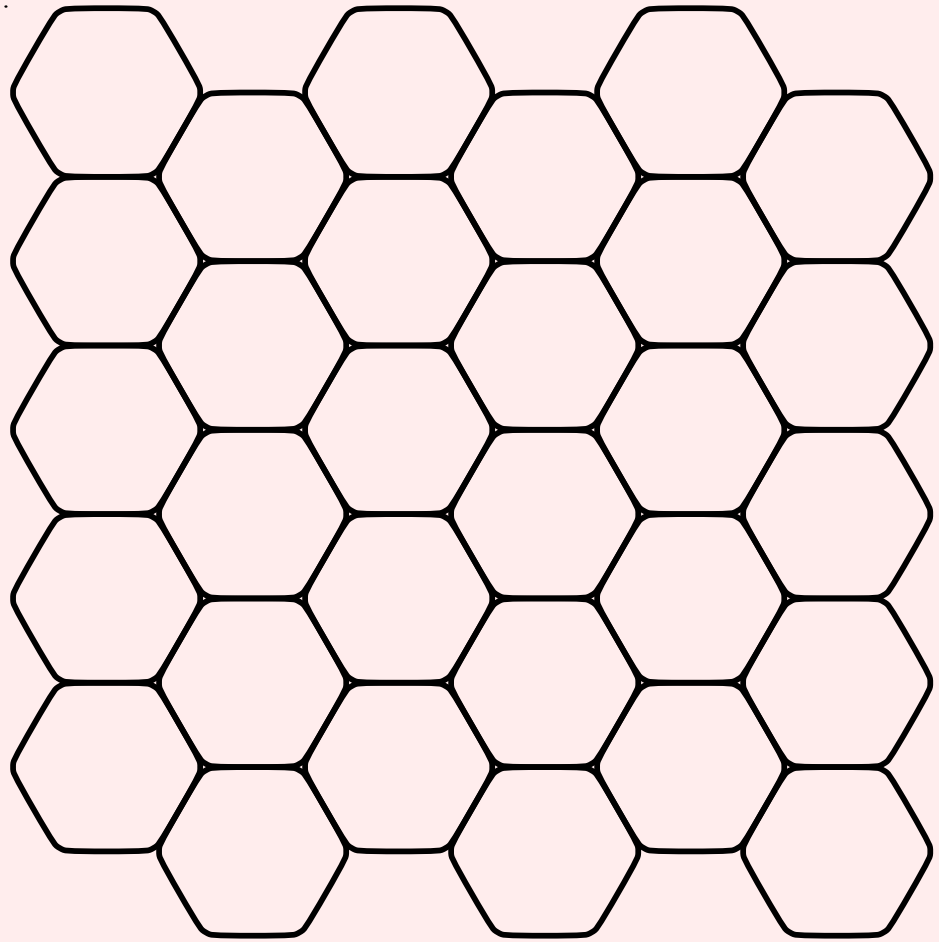
2D bubbles



2D bubbles



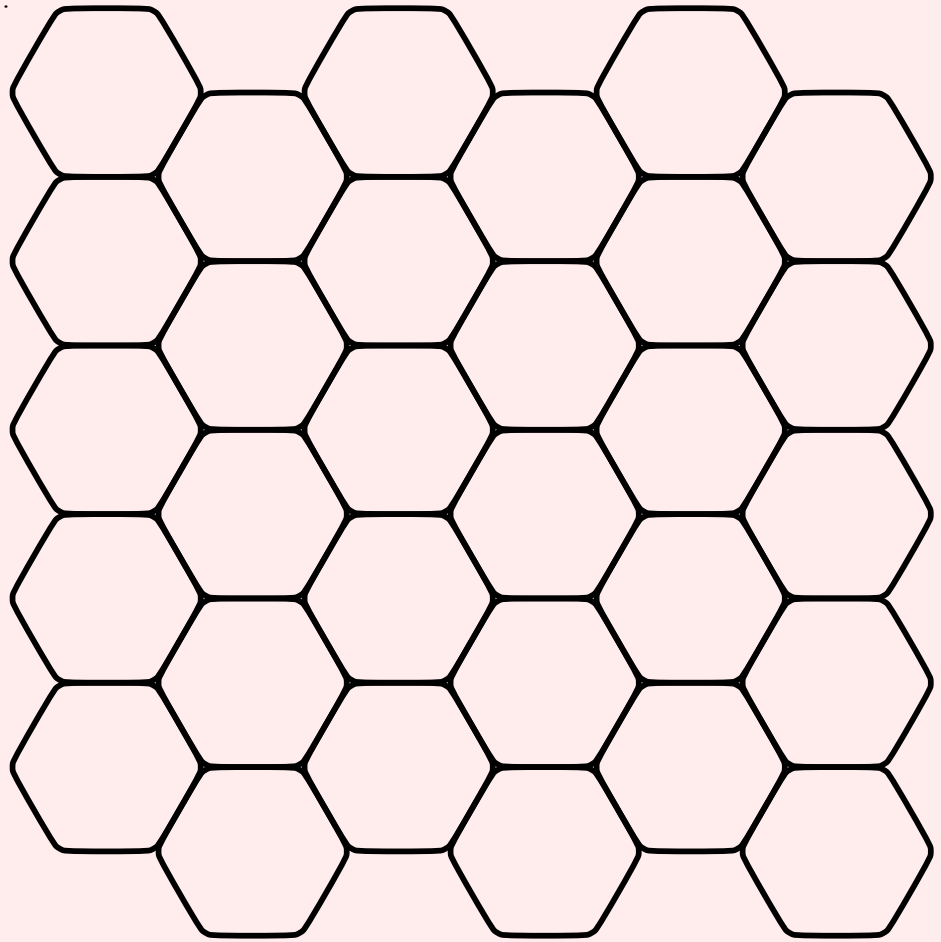
2D bubbles



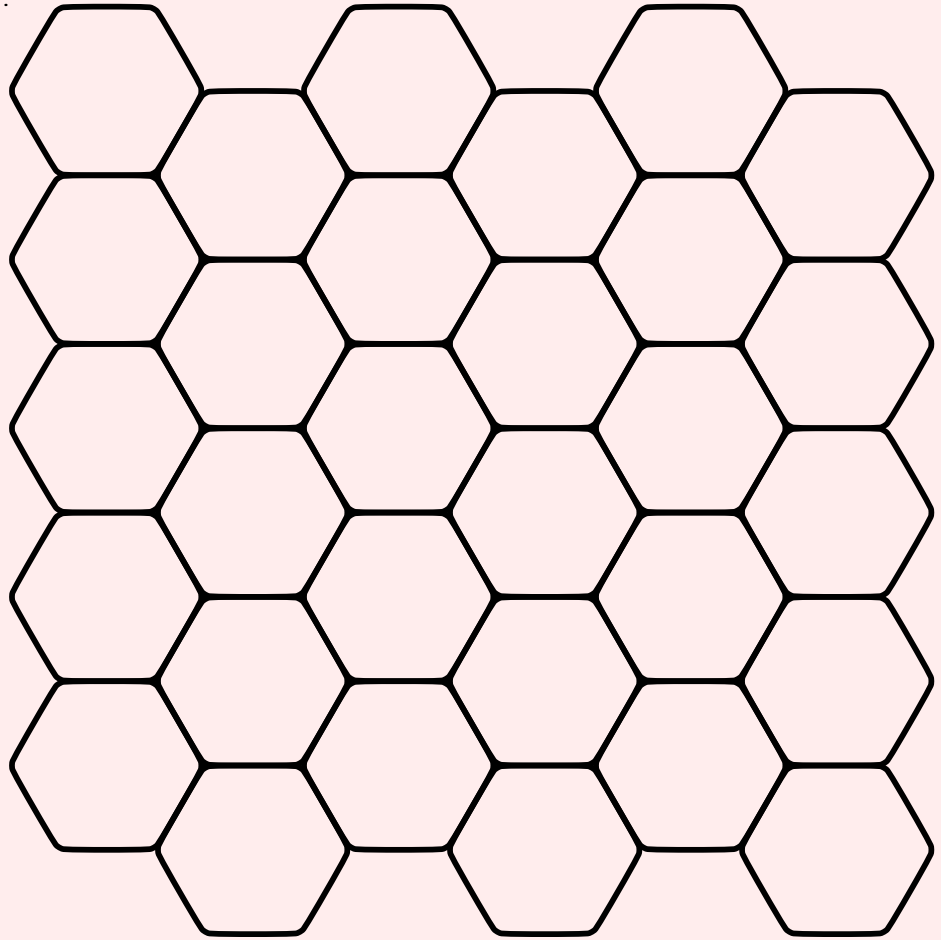
What is the smallest number of circles that can cover more area of a square than a single circle?
An example of the scientific method in action.

Luís Nobre Gonçalves

2D bubbles

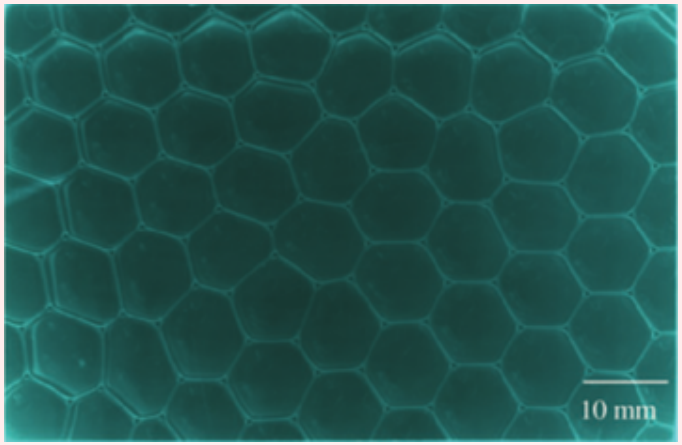


2D bubbles



The hexagonal configuration minimizes “free energy”.

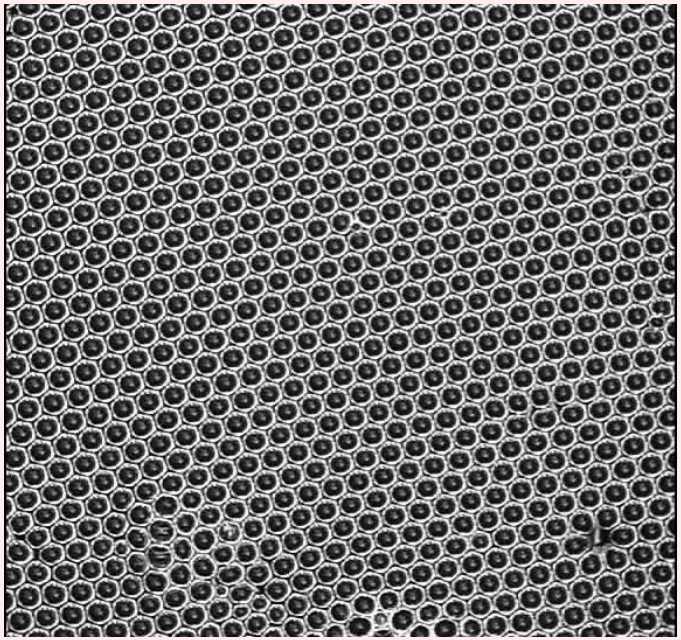
Hexagons in nature



2D foam

red beryl

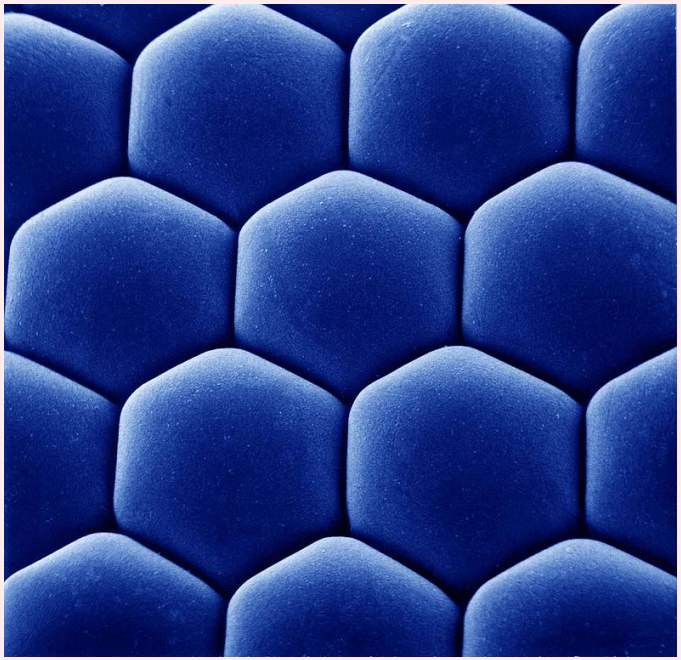
Hexagons in nature



bubble–raft

honeycomb

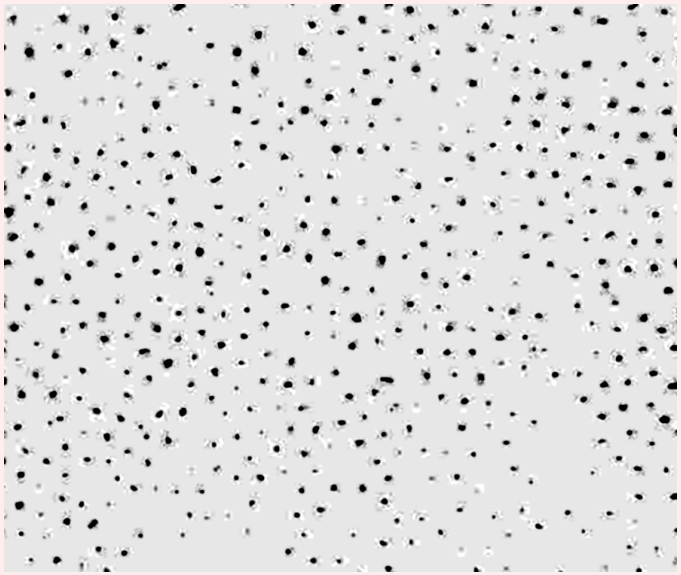
Hexagons in nature



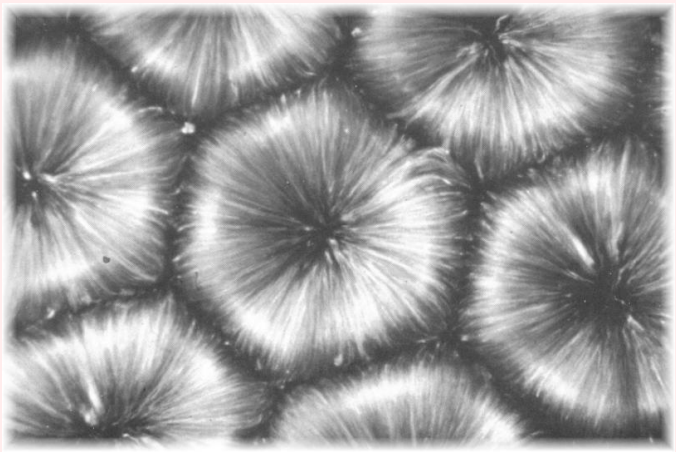
ommatidia

giant's causeway

Hexagons in nature

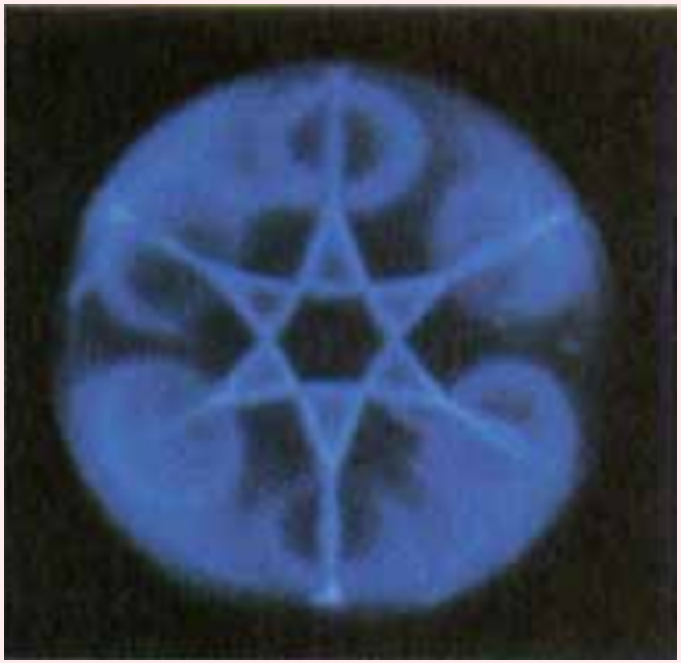


dusty plasma

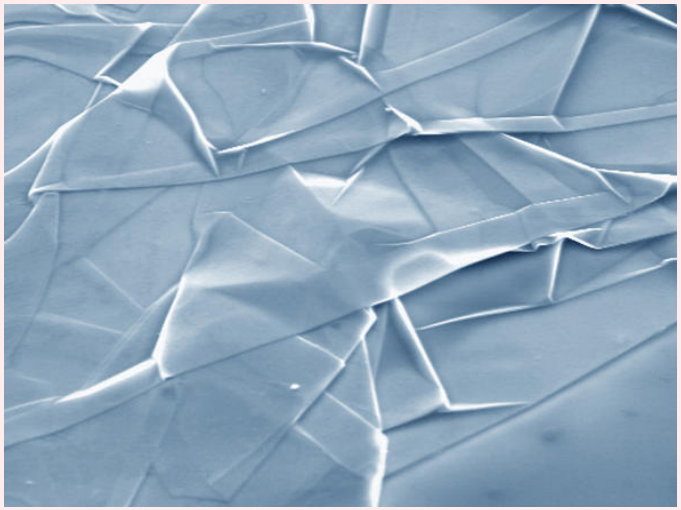


Rayleigh-Bénard convection

Hexagons in nature

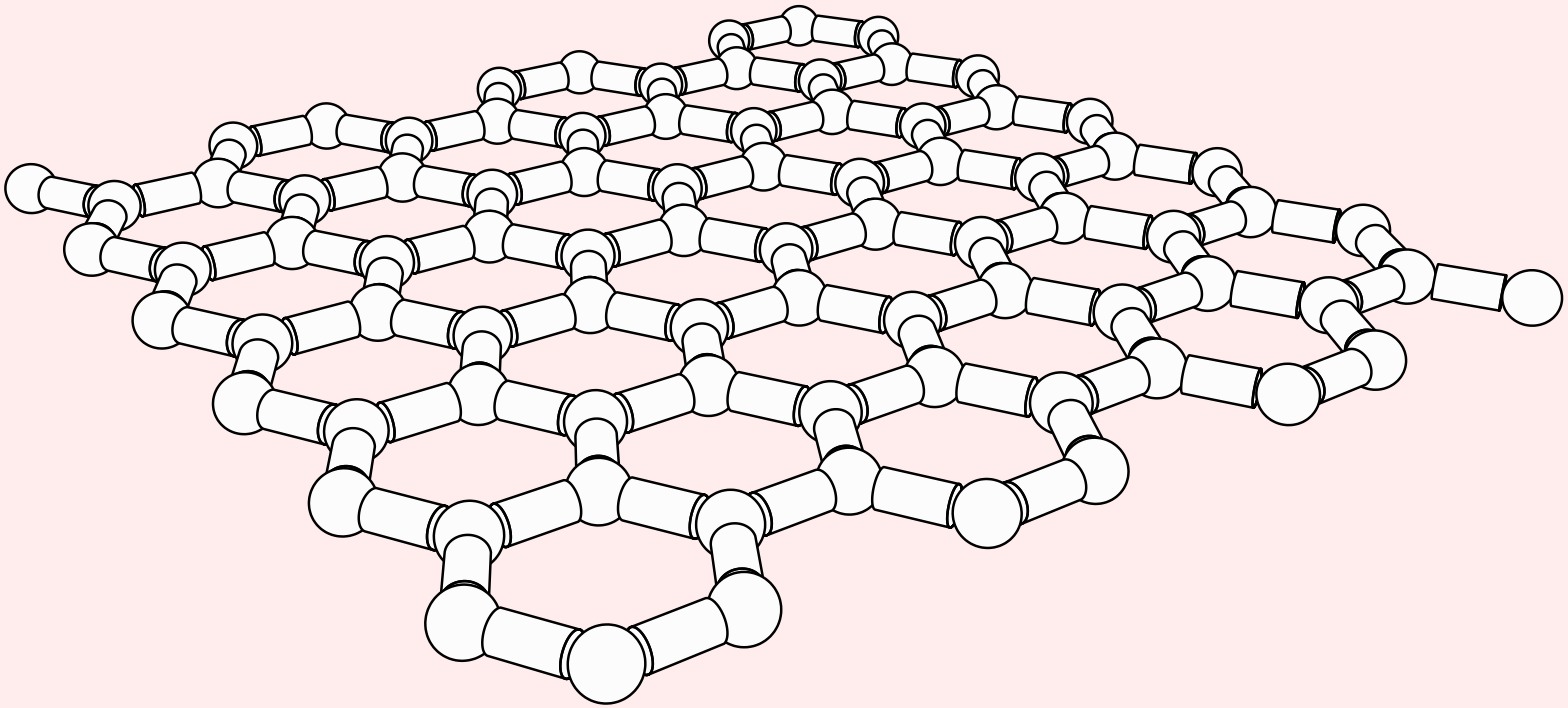


hex. blue phase



graphene

Graphene



Nobel Prize in Physics 2010. Very unusual electronic properties. Will be used in the "Subsea Sensors for Oil and Gas" project.

- Ask the same question but in a different framing.

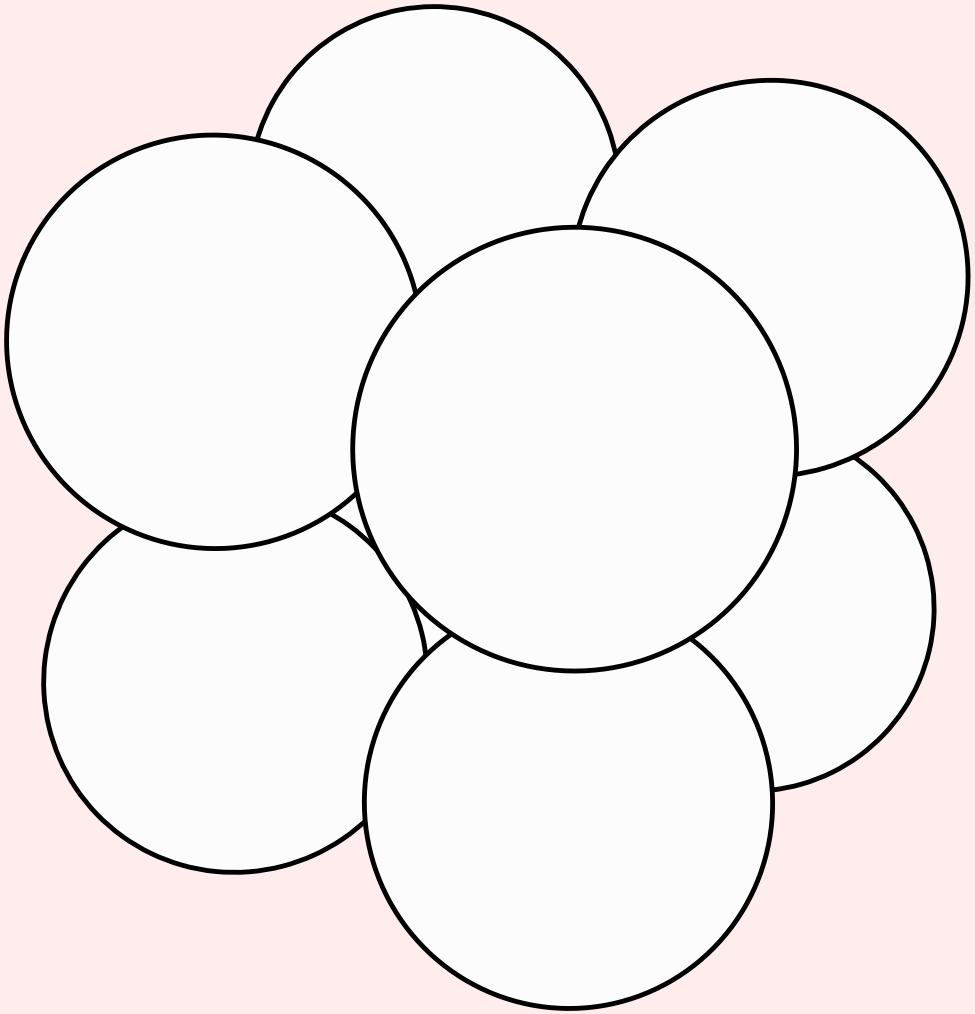
3D

What is the smallest number of **spheres** that can **fill** more **volume** of a **cube** than a single **sphere**?

Cubic configurations

- Cubic
- Boby–centered–cubic (bcc)
- Face–centered–cubic (fcc)
- Clathrate type I ("dfcbcc")

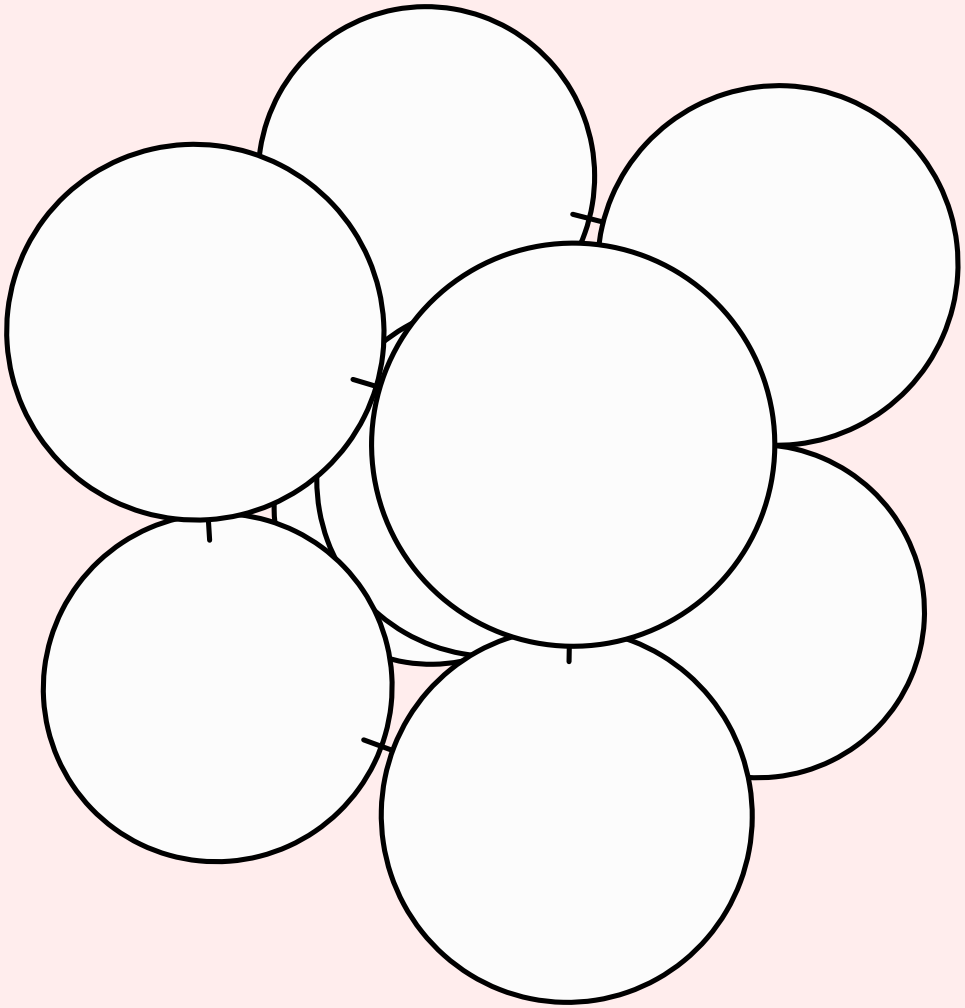
Cubic



$$F_1 = \frac{\pi}{6} \approx 52\%$$

Ex.: ${}_{84}\mathrm{Po}$

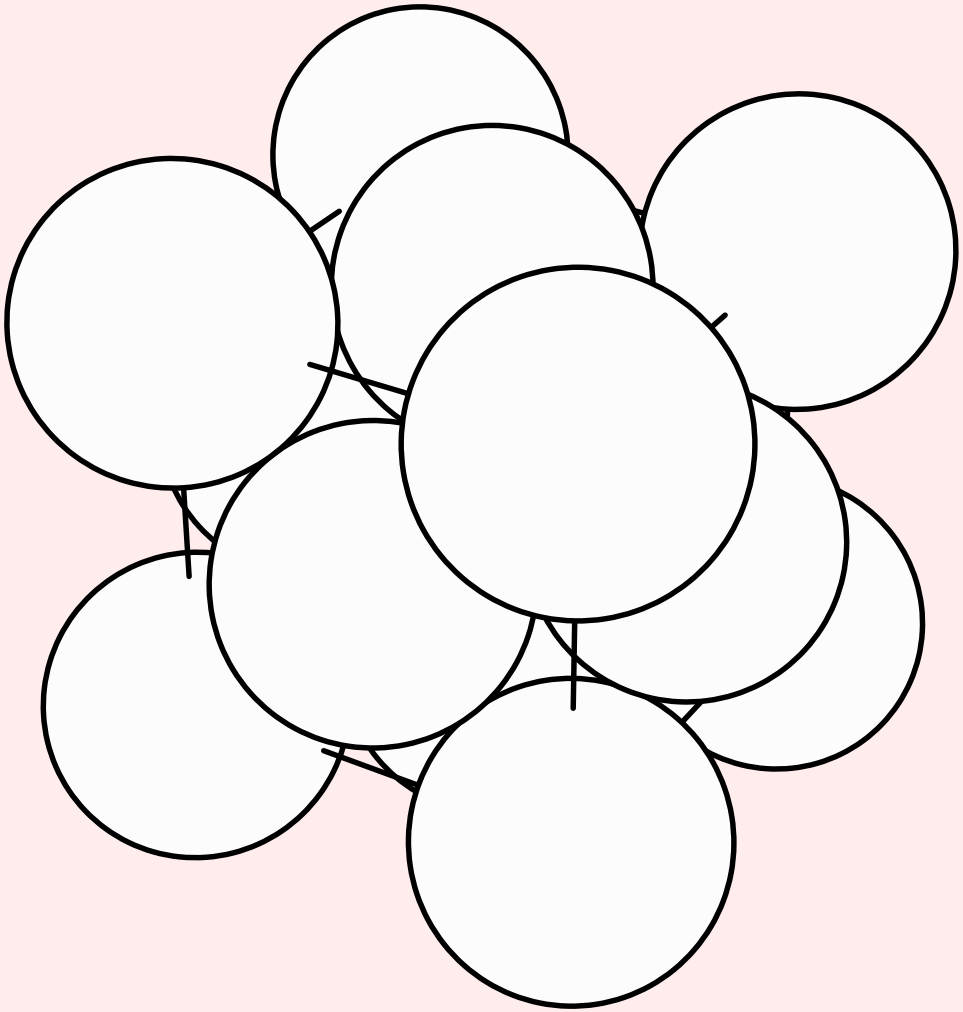
Boby–centered–cubic (bcc)



$$F_\infty = \frac{\pi\sqrt{3}}{8} \approx 68\%$$

Ex.: ${}_{26}\mathrm{Fe}$

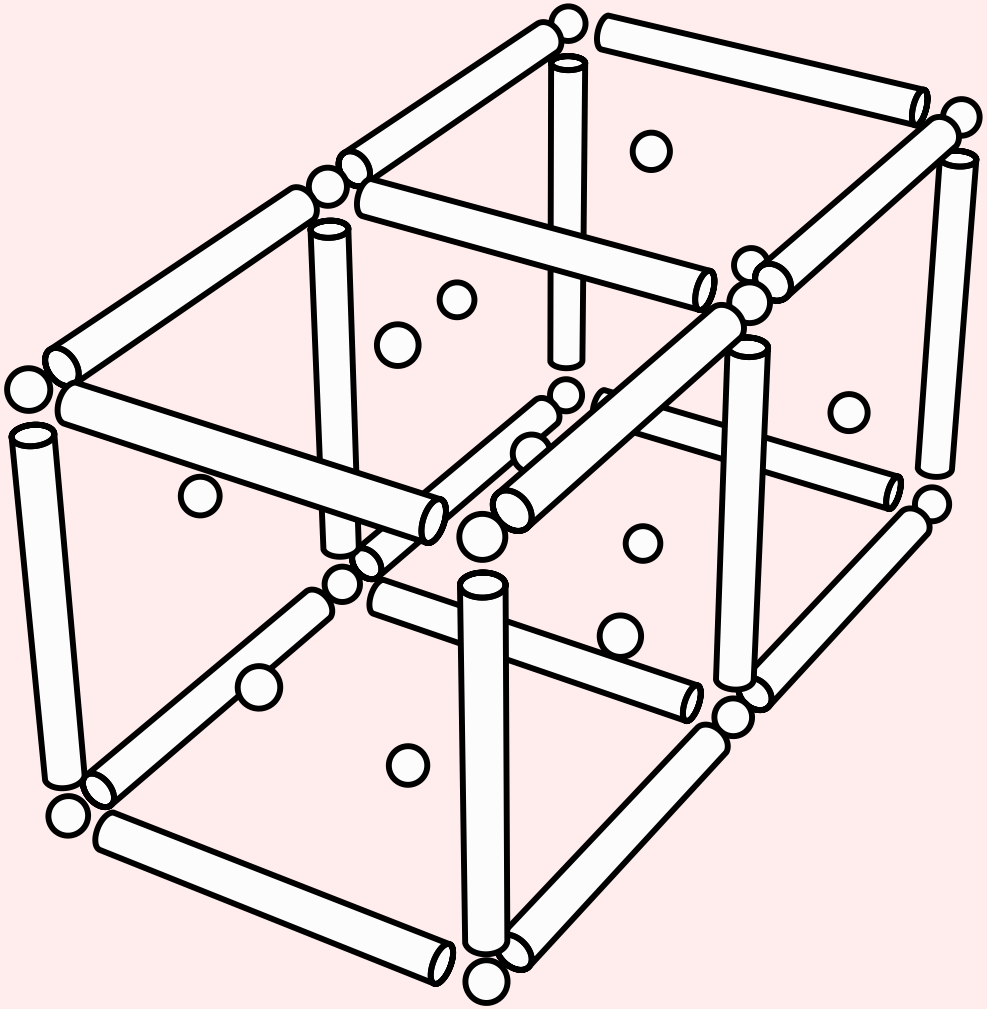
Face–centered–cubic (fcc)



$$F_\infty = \frac{\pi}{3\sqrt{2}} \approx 74\%$$

Ex.: $_{29}\mathrm{Cu}$

Face–centered–cubic (fcc)



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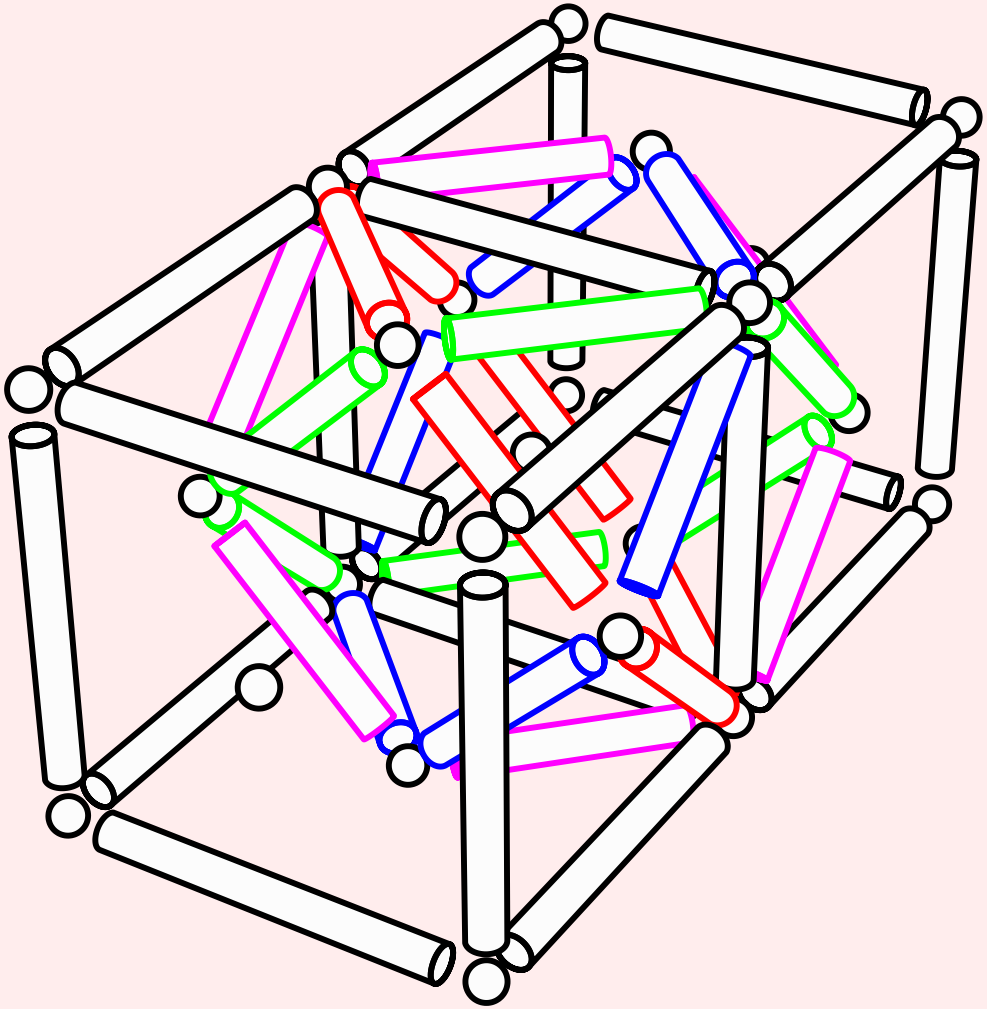
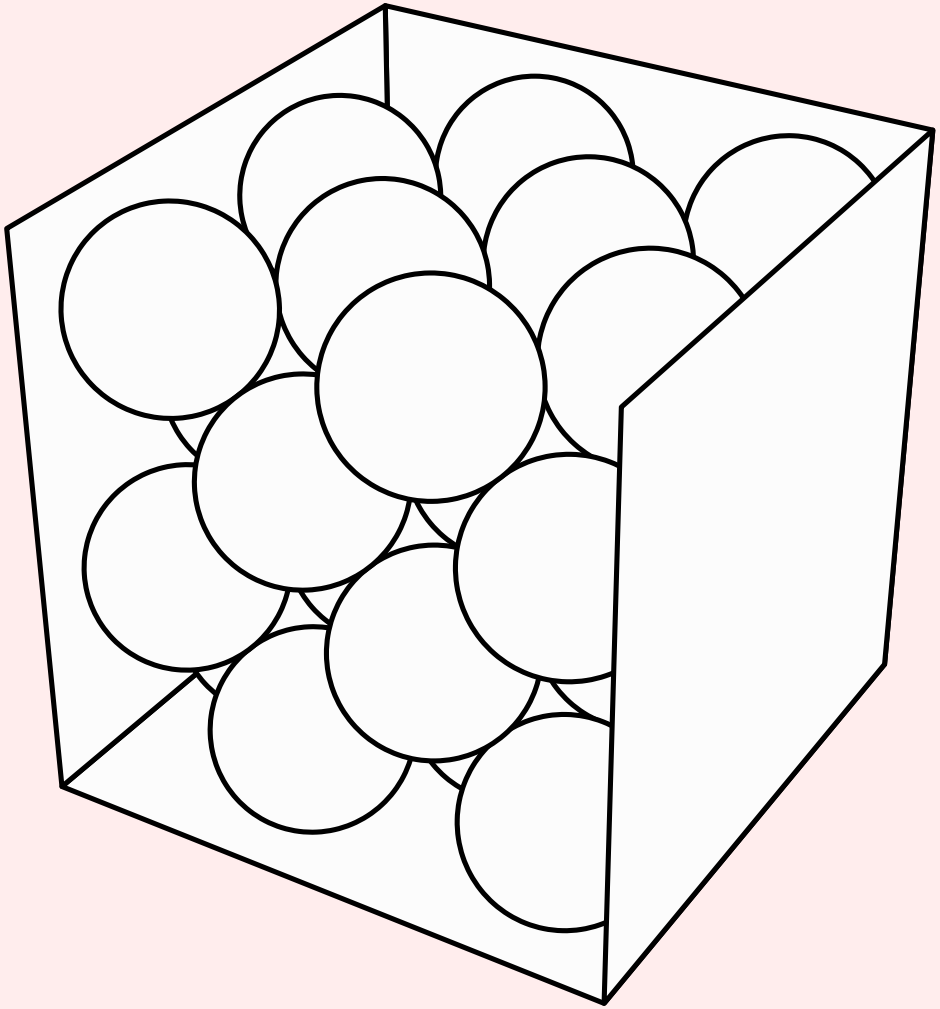
Face–centered–cubic (fcc)



$$F_\infty = \frac{\pi}{3\sqrt{2}} \approx 74\%$$

Ex.: $_{29}\mathrm{Cu}$

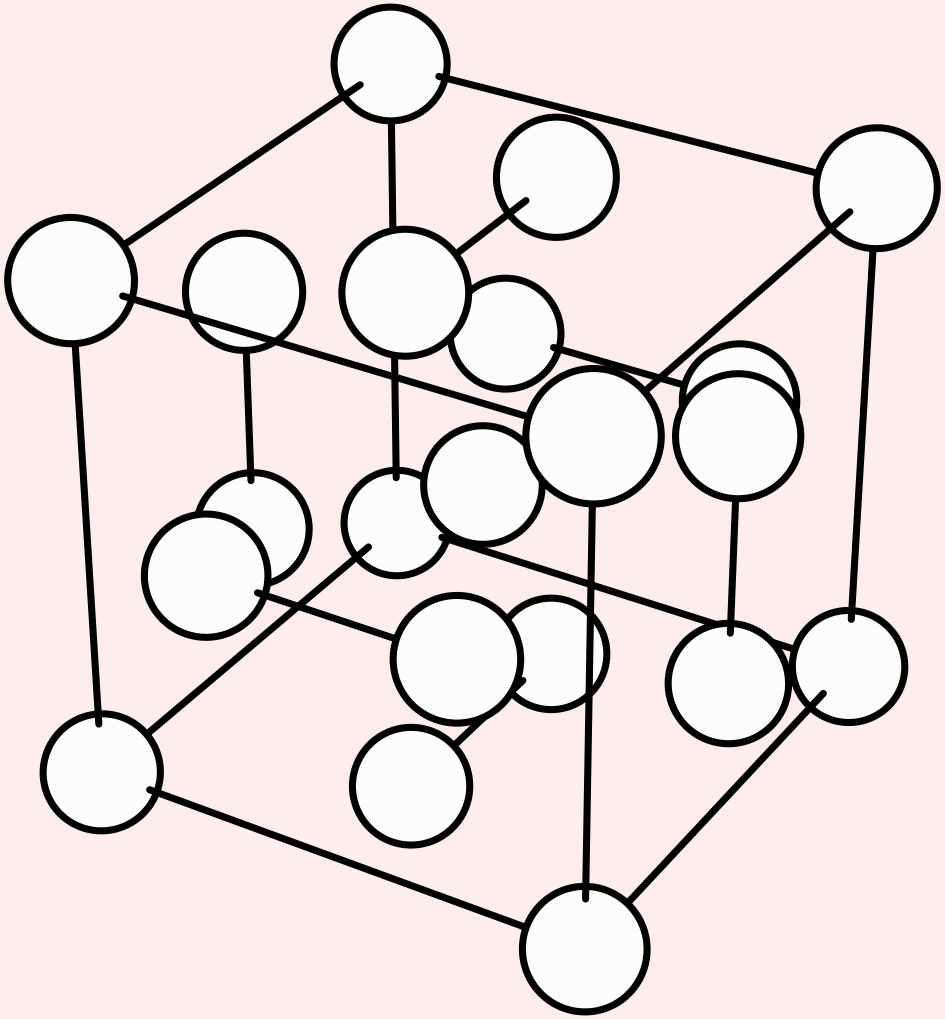
Face–centered–cubic (fcc)



$$F_{31} \gtrapprox 52\%$$

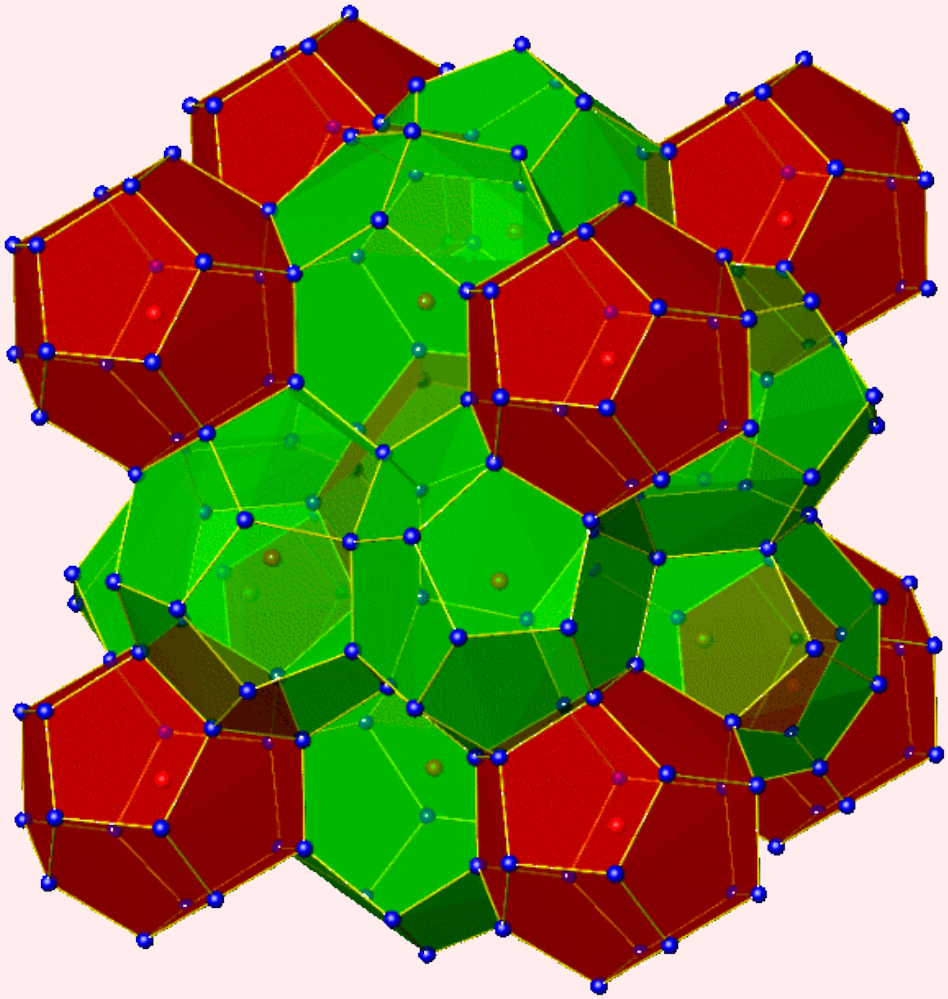
Ex.: $_{79}\mathrm{Au}$

Clathrate



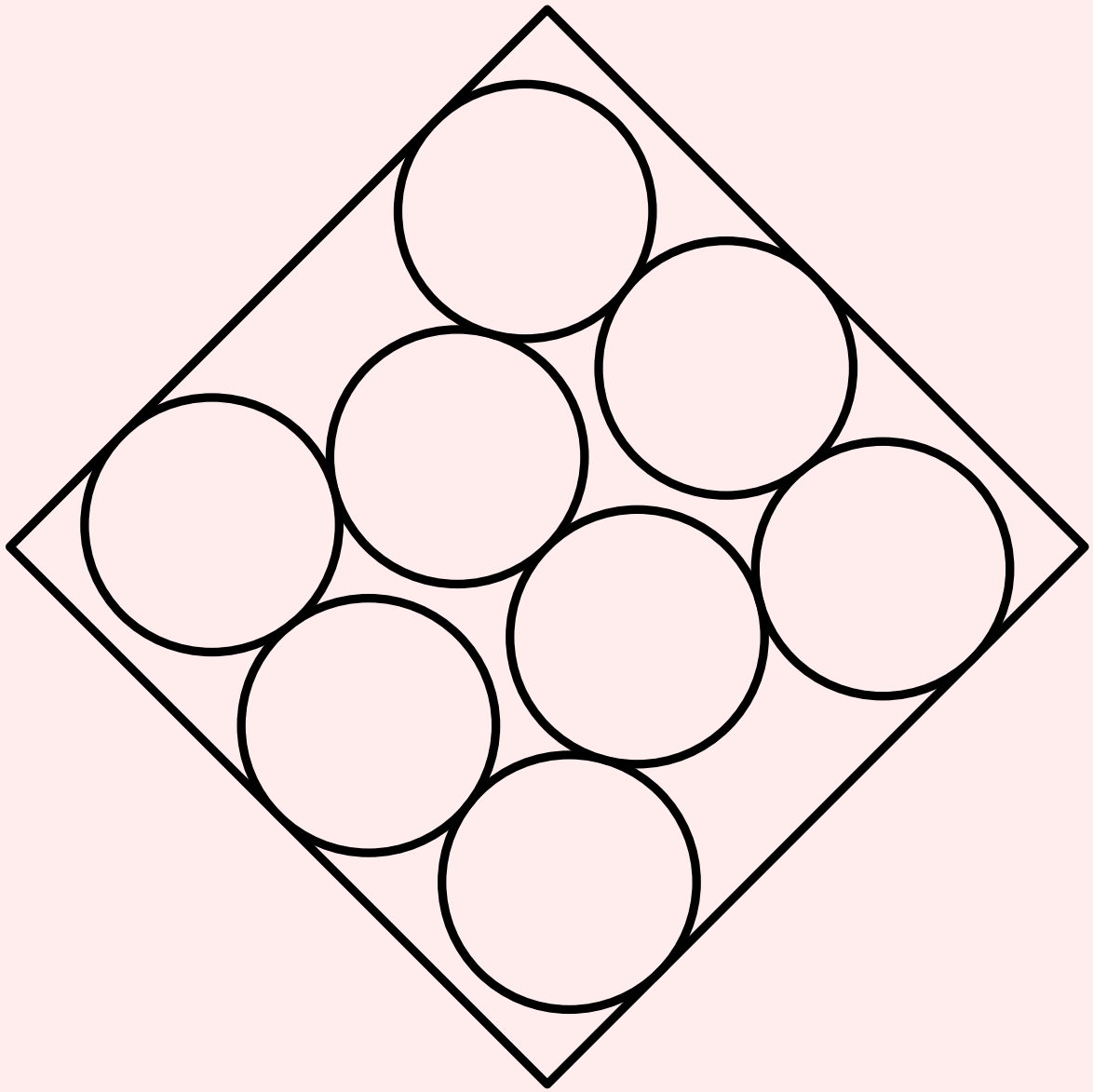
$$F_\infty = F_1$$

Methane Hydrate / Weaire–Phelan



$$F_\infty = 1$$

Summary



Summary

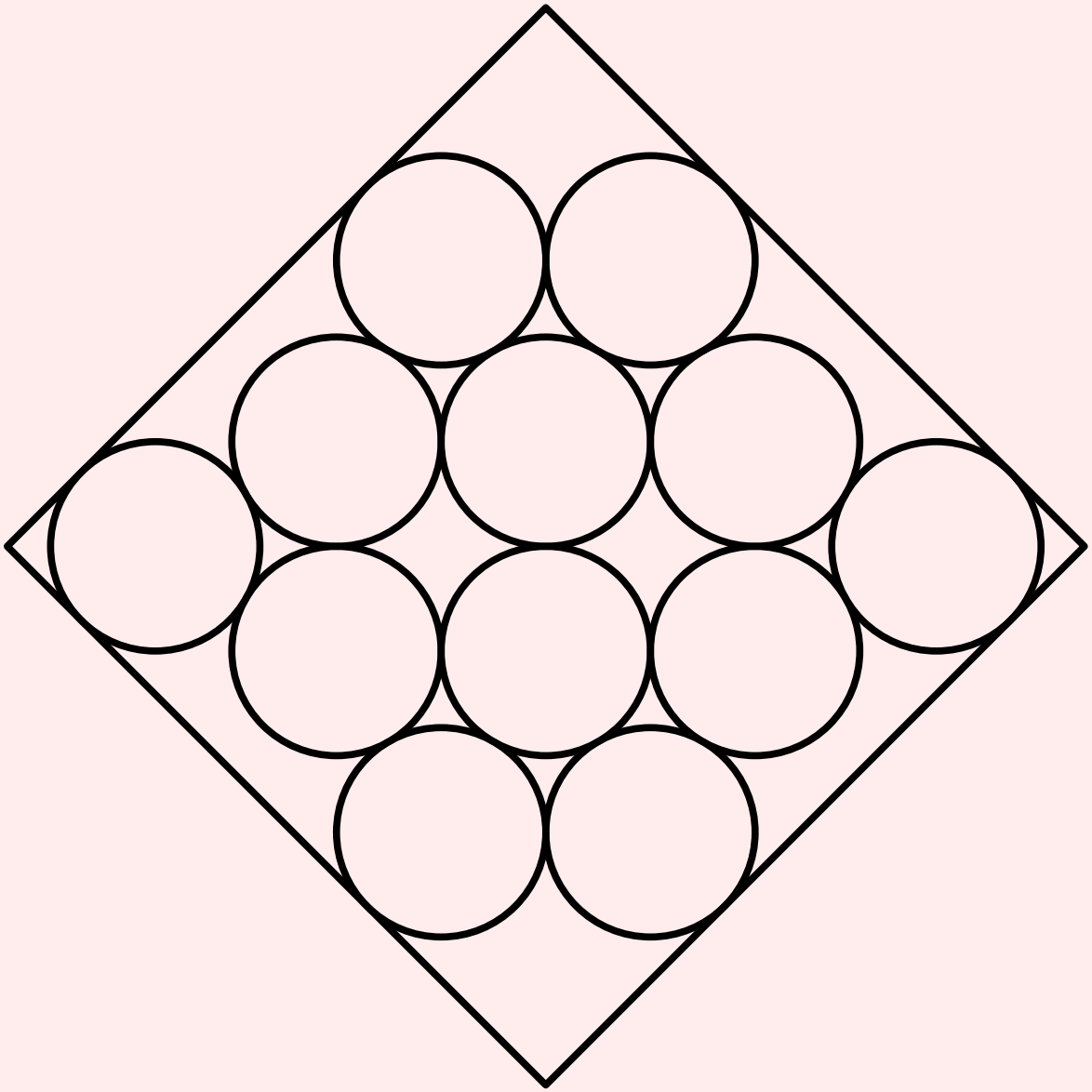
- The SM is the business of questions
- Optimization is always present (together with a reason)
- Know the world

Summary

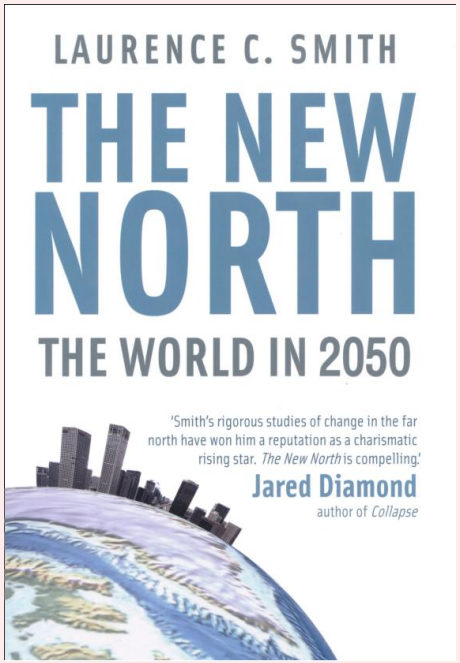
- The SM is the business of questions
- Optimization is always present (together with a reason)
- Know the world

"Luck favors the prepared mind" [Louis Pasteur]

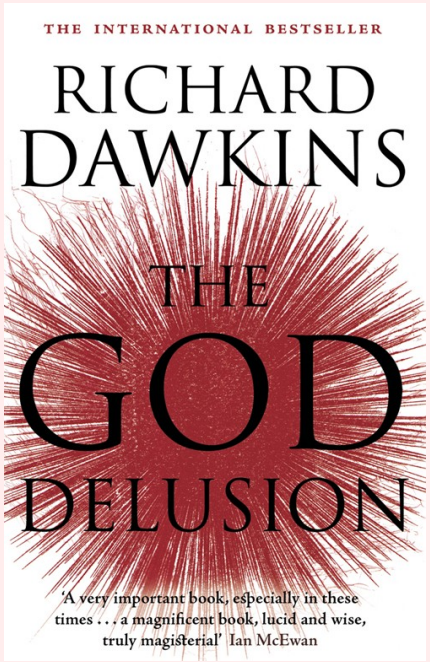
References



Books



978-1-84668-876-8



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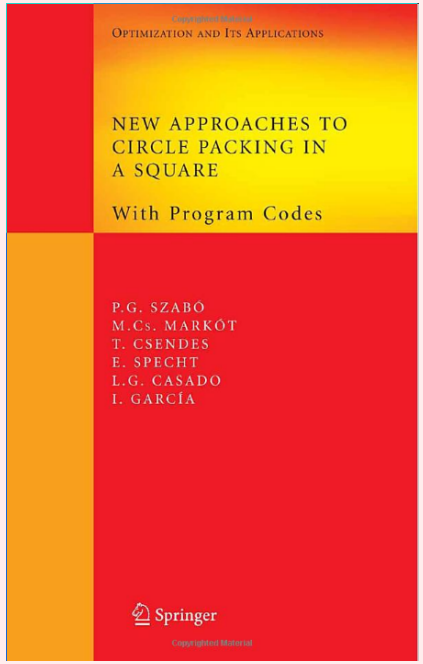


978-0-387-45673-7

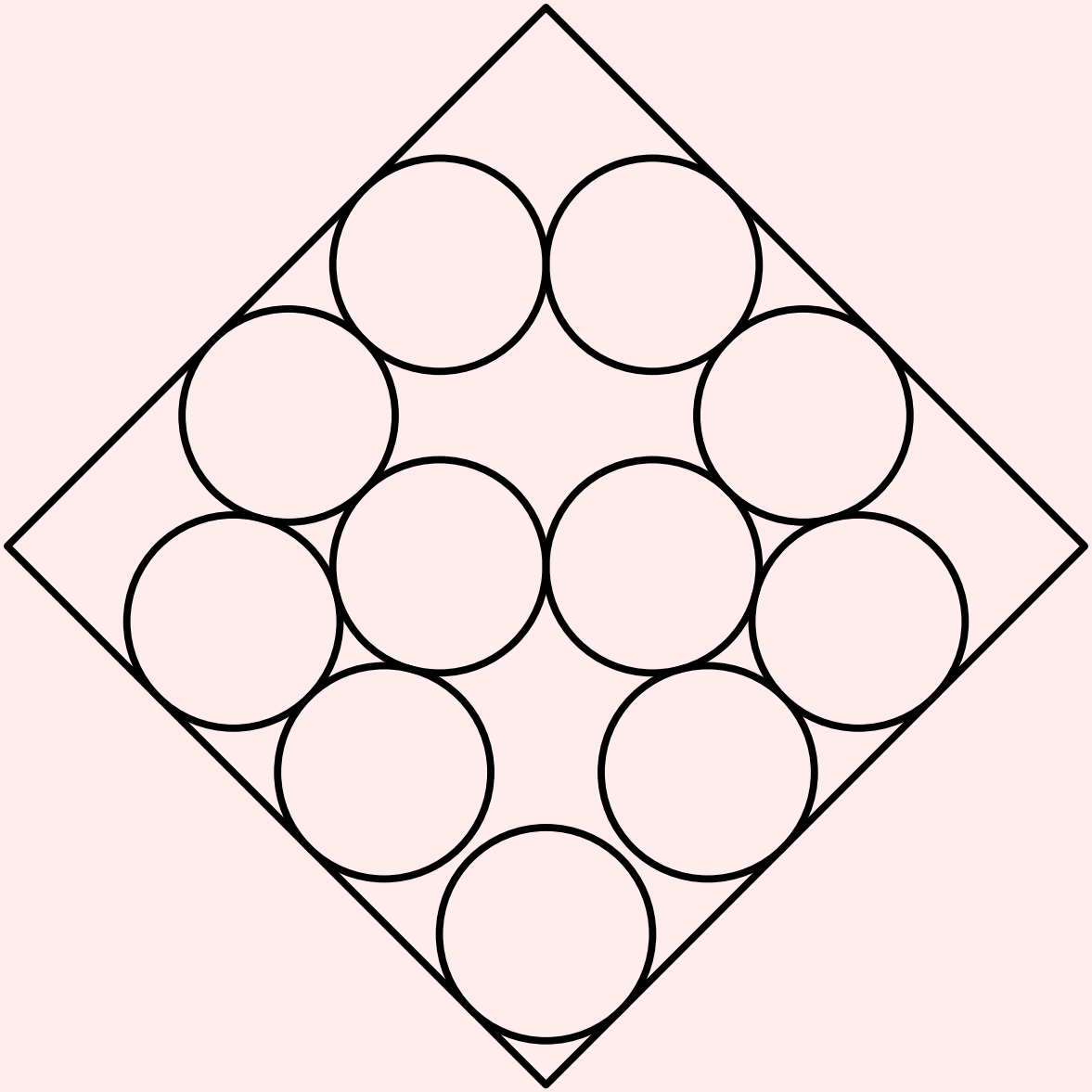
What is the smallest number of circles that can cover more area of a square than a single circle?
An example of the scientific method in action.

Luís Nobre Gonçalves

Links

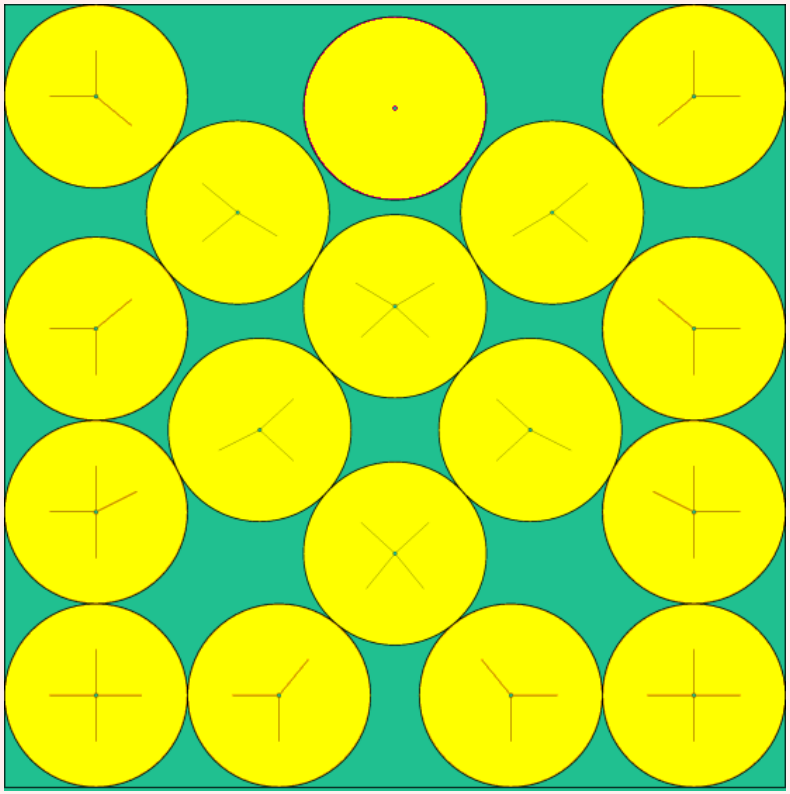
- Definition of “Scientific method” in the `wikipedia`
- The best known packings of equal circles in a square
- Erich’s Packing Center
- What the bees know and what they do not know
- The rise of graphene
- Foam Physics
- Melting snow and ice : a call for action
- Clathrate gun hypothesis
- Avoiding Hydrates in the Petroleum Industry
- Ten Simple Rules for Doing Your Best Research
- “John Cleese on Creativity (video from a training)”
- Faculdade de Ciências e Tecnologia da Universidade Nova de Lisboa

Appendix



Successes and pitfalls of the scientific method

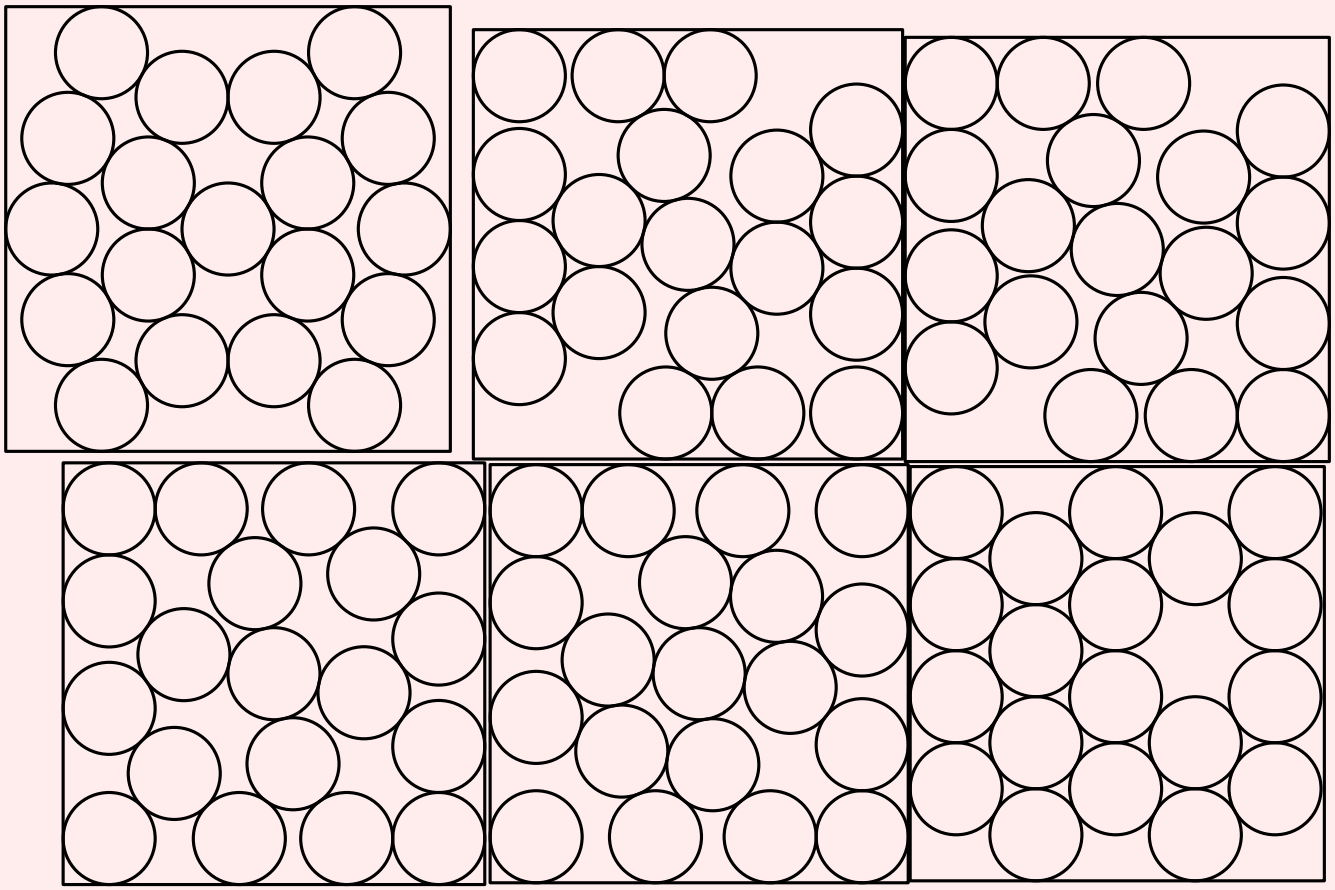
- Vortex mill
- Kevlar^{TM}
- Magnetotherapy
- Cold fusion



$$F \approx 0{,}734$$

Six unknown angles.

19



What is the smallest number of circles that can cover more area of a square than a single circle? An example of the scientific method in action.

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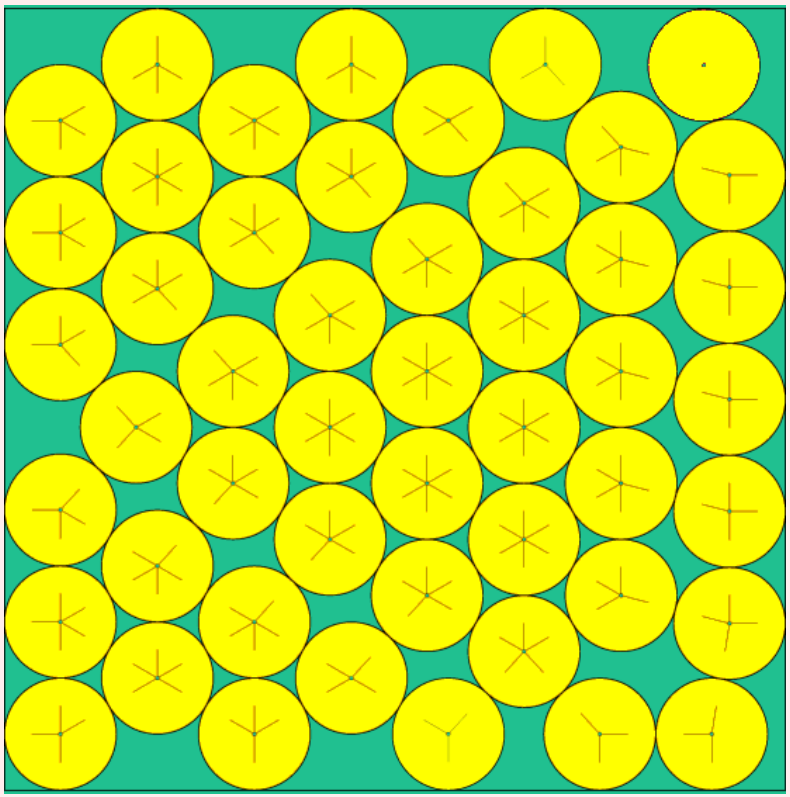
First non-square perfect square



$$F_{49} \approx 0{,}791$$

Acknowledgements

- LaTeX Beamer class
- METAPOST
- GNUPLOT

Further reading links

- The first experimental scientist: Alhazen
- "Dialogue Concerning the Two Chief World Systems" by Galileo Galilei

My quote

"The scientific method is the most powerful tool ever invented and, therefore, it is also the most dangerous tool. As soldiers face death in war, scientists face craziness in their jobs."